

Investigations into Fuzzy Preference Relations, Choice functions and Rationality Conditions: A Decision-Analytic Perspective

Abstract

Decision-making is all pervasive in human activities, and most managerial problem solving is about making proper decisions. A very fertile area of research that uses insights from numerous other disciplines, named Decision Science/Analysis, has emerged as a result of tremendous interest in the art and science of decision-making. In this thesis we consider decision-making situations where the decision-maker is required to choose the best alternative from the set of explicitly available alternatives. In classical social choice theory, preferences of individuals are obtained (0 or 1) for every pair of alternatives. The matrix thus obtained is called *preference relation*. Various *choice functions* give us ways to arrive at a “rational” choice once the crisp preference relation of an individual has been obtained. Ultimately the attempt is to arrive at a social ordering of alternatives by aggregating individual preference relations into a social preference relation. Work in the area of fuzzy choice analysis is based on the premise that preferences are essentially fuzzy memberships and modeling of choice-making processes are more realistic if corresponding notions from crisp social choice theory are “fuzzified” to accommodate fuzzy preferences.

Most of the work done so far in this area is theoretical and largely mathematical. By doing an exhaustive and critical review of related research, we present a large body of research in a digestible fashion. We place a special emphasis on bringing forth the intuitive underpinnings of various notions and making the notions understandable through examples. Some intuitive difficulties associated with the mathematical design of various choice functions are illustrated with examples. We attempt to classify choice functions and rationality conditions proposed in the literature, and establish inter-relationships between/among them based on their intuitive similarities.

Another contribution of this thesis is to bring the use of fuzzy preference relations into the domain of individual decision-making methods. Towards this end we conducted

an experiment to empirically test various notions in a realistic choice situation. Our experiment is the first attempt to test the notions of fuzzy choice theory empirically. We considered a choice situation that required choosing from a set of five two-wheeler vehicles. We elicited the preferences of our respondents over all pairwise comparisons through a graphic user interface-based programme. Analysis of fuzzy preference relations thus obtained was done using statistical analysis package SAS. Analysis of the data shows that, given the freedom, people naturally express their preferences as fuzzy memberships. Also, their preferences are seldom transitive and connected as understood by the commonly used definitions. Performance of various choice functions is assessed based on their predictive efficacy. Three clear clusters emerge from the analysis. Also, most of the intuitive peculiarities of choice functions brought out by us are vindicated by the experimental data.

We observe that the performance of choice functions becomes unstable when the underlying fuzzy preference relation is non-transitive and unconnected. We propose several new choice strategies that improve the quality of choice making process in situations where restrictions such as transitivity and connectedness are violated by the FPRs. Interestingly, it was found that forcing connectedness for half the pairwise comparisons may be a better way to elicit preferences. This is because there was no significant reduction in the efficacy of choice functions and the cognitive effort required by the decision-maker is much less. Some choice functions that were proposed by us (to overcome existing intuitive difficulties) also performed better compared to their previous versions. We observed that FPRs that were obtained from the experiment violated the rationality conditions existing in the literature.

We establish some linkages between conventional multiple attribute decision-making method and fuzzy preference relations which in turn lends credibility to fuzzy choice strategies as viable tools for decision support. These linkages also enable us to understand restrictions (on FPRs) that may be reasonable to expect, thus giving a better idea of pragmatic rationality. It appears more reasonable to expect real-world FPRs to obey acyclicity rather than transitivity. This belief is substantiated by the finding that all the FPRs obtained in experiments were acyclic but very few were transitive.