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Cost of Equity and Leverage under "Fair" Rate-of-Return Regulation

by

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A rate-of-return regime characterized by "fairness" satisfies two criteria: the total allowed return on the rate base is equal to the cost of capital, and the regulated firm should be able to raise capital without either gain or loss to existing equity holders. Assuming a monopoly firm with a single product, a single-period state-preference world, risk-free debt, corporate (but not personal) tax, and perfectly price inelastic demand; this paper shows that in such a regime the tariff of a regulated firm will have to be reset with leverage. This resetting arises because unlike in a Modigliani-Miller world where firm (and equity) value is enhanced by the leverage (because of interest tax shields), "fairness" implies that interest tax shield benefits accrue to consumers, after ensuring equity holders receive a return on equity commensurate with systematic risk.

If demand is not perfectly inelastic, a change in tariff consequent upon change in leverage will also lead to a change in equilibrium output. With the assumption that there is a single driver of systematic risk (that of output), the cost of equity-leverage relationship obtained with perfectly inelastic demand is shown to still hold.

Key words: regulation, cost of equity, leverage, price-elasticity

JEL Classification: G38, L51

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1. INTRODUCTION

The central focus of this paper is the relationship between cost of equity and leverage of a firm operating in a rate-of-return regulatory regime. Literature in this field can be divided into two broad categories–the first focusing on firm value and leverage, the second focusing on incentives provided by regulatory practice.[1] However, standard cost of equity-leverage relationships, such as the Modigliani-Miller (MM) or the Miles-Ezzel formulations (see Taggart 1991) do not incorporate the concept of 'fairness' (as defined in Greenwald 1980). Also there exists no closed form valuation model that addresses the Jaffe and Mandlekar (1976) concern that the valuation formula for a regulated firm requires specification of its demand and supply curves.[2] In this paper I intend to partly fill this gap by deriving the cost of equity-leverage relationship for a firm with price-elastic demand, operating in a regulatory environment that assures "fairness".

I will assume a single-period state-preference world, risk-free debt and corporate (but not personal) tax. These assumptions will help me focus on the following central insight of this paper. In an unregulated firm, the totality of value (including the present value of income tax) is unchanged by leverage. Three sets of agents (lenders, equity investors and the government) participate in this total value in the Modigliani-Miller (1963) world. The value of the firm itself (lenders and equity investors) increases with leverage because of increase in interest tax shields. However, in a regulated world the totality of value of these three sets of agents is affected by leverage if the rate of return set by the regulator is "fair."

A rate of return is "fair" (Greenwald 1980, 360-363) if it satisfies two criteria: the total allowed return on the rate base is equal to the cost of capital, and the regulated firm should be able to raise capital without either gain or loss to existing equity holders. These two criteria together imply that any investment should result in an equal increase in the value of the rate base. A number of admissible classes of rate bases (including historical cost) can satisfy this definition of "fairness".

Given "fairness" the totality of value may also require the measurement of value changes of two others sets of agents—consumers and input suppliers—in addition to the three sets of agents mentioned above. For instance, consider a regulated firm, subject to corporate tax, that faces a perfectly inelastic demand, and whose cost of equity is higher than the cost of debt. As I shall show subsequently, under the criterion of fairness, a higher proportion of equity relative to debt will imply that customers face higher tariffs (with no

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[1] While the boundary between these two categories is blurred, the first category includes Brigham, Gapenski and Aberwald (1987), Clarke (1980), Elton and Gruber (1971 and 1972), Gordon (1967), Gordon and MacCullum (1972), Jaffe and Mandlekar (1976), Litzenberger, Ramaswamy and Sosin (1980); the second includes Binder and Norton (1999), Fraja and Stones (2004), Spiegel (1994), Spiegel and Spulber (1997) and Taggart (1981 and 1985).

[2] The generalized formula provided by Jaffe and Mandlekar (1976) finesses the issue of supply and demand curves.

change in firm output given the assumed elasticity) and consumers will therefore lose value. The implication of this insight is that, unlike in an MM world, the risk-adjusted discount rate of the gross revenue stream (r_{REV}) enters the valuation model. Additionally, if the demand is not perfectly price inelastic, a change in tariff will also lead to a change in output and the risk-adjusted discount rate of the cost stream (r_{COST}) will also enter the valuation model. Therefore, the cost of equity–leverage relationship of a rate-of-return regulated firm will reflect the risk-adjusted discount rates of both the revenue and the cost streams. I will show that, in general, the cost of equity will increase with leverage at a steeper rate than with the MM formulation.

This article focuses exclusively on the third step of the four-step rate-making process described by Spulber (1989, 274-279)—this is the choice of the allowable rate of return. It finesses the other steps: calculation of costs, calculation of the rate-base and design of the rate structure.

Given the assumption that debt is risk-free my interest is in the cost of equity. I will use a rate of return framework that treats the initial invested equity at book value as the rate-base. The tariff set will cover the regulator assigned post-tax return on this equity[3], as well as the cost of debt, operating expenses and depreciation. Such a rate of return regime can alternatively be specified as a weighted average cost of capital on total capital (debt and equity) employed and belongs to the class of admissible rate bases that assure "fairness". I will however use an assigned return on equity approach.

My starting point will be the unlevered (all-equity) regulated firm that has, in some fashion, resolved its production-investment decision. The firm invests at time 0, operates for one period and has no salvage value at the end of this period. The regulator uses the initial book value of equity as the rate base and assigns an accounting rate of return on this rate base, consistent with the production-investment decision (assignation of the rate of return also effectively sets the tariff). In a single-period model with fairness this implies that the firm earns a return on equity (consistent with systematic risk of equity in a CAPM world) on the market value of equity. This will ensure that the market and book values of equity are equal, thereby satisfying the Greenwald (1980, 91) requirement of fairness. The core issue addressed in the paper is—how does the allowed return on equity change with leverage, given a regulator who responds to changing leverage by assigning a rate of return on equity that is still "fair." Establishment of a relationship between the systematic risk of the unlevered firm and micro-economic variables[4] is not my primary focus. However, towards the end I will also indicate how the production-investment decision of an unlevered firm can be plausibly made.

The rest of the paper is organized as follows. The cost of,equity-leverage relationship given perfectly price inelastic demand is derived in Section 2 for a firm whose equity

[3] This is essentially the tariff regime in India (CERC, 2001, 10 and 32), "Return on equity shall be computed on the paid up and subscribed capital and shall be 16% of such capital."
"Tax on income from core activity of the Generating Company, if any, is to be computed as an expense and shall be recoverable by the Generating Company from the beneficiaries."
[4] As in Subrahmanyam and Thomadikis (1980), Conine (1983), Booth (1991) or O'Brien (2005)

beta reflects a single driver of systematic risk. Section 3 shows that this relationship is valid for a more general price elasticity of demand. Section 4 provides an illustration of the resolution of the production-investment and cost of unlevered equity-tariff decisions. Section 5 concludes the paper.

2. THE COST OF EQUITY GIVEN PERFECTLY INELASTIC DEMAND

The required cost of equity-leverage relationship is built on the assumptions below (with assumptions labeled A1-A7 held throughout the article.)

A1 Cost and output uncertainty are modeled in a single-period time-state preference framework. A complete set of contingent security prices exists. Given these and a representative individual with quadratic utility, the CAPM can be used to specify security prices (Booth, 1984). While the CAPM will be extensively used in this paper, Section 5 will provide a numerical illustration in a state-preference framework.

A2 The firm faces a corporate tax rate T, but investors are not taxed at personal level.

A3 The firm is a monopolist with a single product.

A4 There is no information asymmetry between the firm and the regulator.

A5 Any debt used is risk-free.

A6 The initial capital investment I at time 0 is a given. There is no uncertainty about the capital cost or depreciation tax shields. There is no salvage value, and depreciation DEPR=I.

A7 All operating cost is variable. Output is state-contingent, but unit variable cost is state-independent. The firm has a state-contingent variable cost as follows:

$$COST = c * Q(s)$$

where c is the state-independent unit cost and Q(s) the state-contingent output.

With this assumption, the source of underlying risk in the firm is the systematic risk (β_{OUTPUT}) of output—the systematic risk of cost (β_{COST}) will equal the systematic risk of output. The regulator assigned unit-tariff is also assumed state-independent. As a consequence, state-contingent revenues will reflect state-contingent output. The systematic risk of revenue (β_{REV}) will, therefore, equal that of output.

The expected operating cost E(COST) is known, as is the systematic risk of the output

A8 Demand is perfectly price-elastic. This assumption is lifted in subsequent sections.

With these assumptions the sequence of decisions is as follows:

In iteration-1, the firm will only use equity. At the outset the regulator and the firm agree on the capital investment "I" to be made at time t=0 (with book value of equity E=I), the expected variable cost (based on the expected output) and the systematic risks of the output and variable cost stream. The regulator assigns an accounting return on book equity, and consistent with this, a state-independent unit-tariff. The firm makes the investment at t=0. At time t=1, the firm adjusts the output to the realized state of the world, and then liquidates.

Iteration-2 is identical to iteration-1, except that the firm indicates at the outset the amount of risk-free debt "D" that it proposes to employ. These iterations are detailed below, with a brief sub-section on the lazy/unfair regulator sandwiched between.

Iteration-1: The Regulator's Decision-The Unlevered Firm

The regulator determines the accounting return on the book value of equity ROE_U and the state-independent unit-tariff P_U, such that the expected revenue $E(REV_U)$ will cover the expected variable cost E(COST) and other charges (OC–comprising depreciation DEPR and a post-tax accounting return on equity (ROE_U) on the initial book value of equity.) ROE_U is based on the principle of "fairness", consequently the market value of the firm will be I, the market value of equity will equal the book value E, and the cost of equity (reflecting systematic risk) will be equal to the accounting rate of return, ROE_U. This tariff results in a state-contingent revenue with an expected value $E(REV_U)$ and systematic risk β_{REV} (with β_{REV} and β_{COST} equal to β_{OUTPUT}).

The expected revenue is:

$$E(REV_U) = E(COST) + OC_U \qquad (1)$$

where other charges OC_U is given by

$$OC_U = I * \frac{ROE_U}{(1-T)} + DEPR = I * \frac{\left(r_F + (r_M - r_F) * \beta_U\right)}{(1-T)} + I \qquad (2)$$

In equation (2) above, as a consequence of the principle of "fairness" the CAPM expected equity return can be substituted for ROE_U (this also ensures that the NPV is zero). The systematic risk is given by equation (3) below.

$$\beta_U = \left[\beta_{REV} * PV(REV_U) - \beta_{COST} * PV(COST)\right] * {(1-T)}/{I} \qquad (3)$$

Substituting (1) and (2) in (3), the systematic risk of unlevered equity can be obtained as a function of model inputs.

$$\beta_U = \frac{\left[(\beta_{REV} - \beta_{COST}) * PV(COST) * {(1-T)}/{I} + (R_F + 1 - T) * {\beta_{REV}}/{(1 + r_{REV})}\right]}{\left[1 - (r_M - r_F) * {\beta_{REV}}/{(1 + r_{REV})}\right]}$$

$$\beta_U = \frac{\left[(R_F + 1 - T) * {\beta_{REV}}/{(1 + r_{REV})}\right]}{\left[1 - (r_M - r_F) * {\beta_{REV}}/{(1 + r_{REV})}\right]} \qquad (4)$$

E(REV) can be obtained from (1), and with this expected revenue and equity beta the value of the unlevered firm (V_U) will be "I".

$$V_U = \frac{E(\widetilde{CF}_U)}{(1 + \mathrm{ROE}_U)} = I$$

where the after-tax period-end cash flow of the unlevered firm is:

$$E(\widetilde{CF}_U) = [E(REV_U) - E(COST)] * (1 - T) + \mathrm{DEPR} * \mathrm{T} = (ROE_U * I) + \mathrm{DEPR}$$

The Levered Firm with a Lazy/Unfair Regulator

If the firm instead used debt D initially and the regulator continues to provide ROE_U, on the actual equity base[5] the change in value of the levered firm will be:

$$\Delta V = -\left[\frac{ROE_U}{(1-T)} - r_F\right] * D * \frac{(1-T)}{(1 - r_{REV})} + \left[\frac{r_F * D * T}{(1 + r_F)}\right] \qquad (5)$$

This is essentially in an APV (Adjusted Present Value) framework. The first term in the above equation reflects the after-tax present value diminution of the lower tariff paid by customers. The tariff is lower since the regulator will permit only the risk-free return on the amount of debt, D; and not the return on equity (grossed up by the corporate tax). The second term is the present value of the tax shield on debt interest.

If the systematic risk of equity is positive (so that the cost of equity is higher than that of debt) the equity holders suffer a value reduction by this introduction of debt. What is of interest is the appearance of the expected risk-adjusted discount rate of revenues and,

[5] I am ignoring the possibility that the regulator leaves the tariff unchanged thereby providing equity holders a windfall gain (see Sherman (1977)).

therefore, the beta of revenue in the determination of this value. The present value decline will be at the r_{REV} corresponding to the revenue stream and not, as in Elton and Gruber (1971), at the cost of unlevered equity.

Iteration 2: The Regulator's Decision-The Levered Firm

However, my concern is with a regulator who adjusts the return on equity "fairly". If the regulator allows a return on equity ROE_L on the revised book equity that reflects leverage, then the appropriate cost of equity can be derived from the following the equations. Equations (6) and (7) are the flow-to-equity valuations of the unlevered and levered firm, respectively; equation (8) is the APV condition for the levered firm to have the same value as the unlevered. Equation (8) reflects that the present value of the incremental revenue stream less the present value of the incremental cost stream together with the present value of interest tax shield is zero.

$$\frac{\left[\left|E(REV_U)-E(COST)\right|*(1-T)+DEPR*T\right]}{(1+ROE_U)}=I \tag{6}$$

$$\frac{\left[\left|E(REV_L)-E(COST)-r_F*D\right|*(1-T)+DEPR*T-D\right]}{(1+ROE_L)}=I-D \tag{7}$$

$$\left|E(REV_L)-E(REV_U)\right|*\frac{(1-T)}{(1+r_{REV})}-\left|E(COST_L)-E(COST_U)\right|*\frac{(1-T)}{(1+r_{COST})}+\frac{r_F*D*T}{(1+r_F)}=0 \tag{8}$$

By substituting REV_U and REV_L in equation (8) using equations (6) and (7), we obtain the following relationship between the cost of equity and leverage to maintain fairness (ensuring that ΔV is zero), for a leverage L=D/I. Note that with perfectly inelastic demand the value of the second term (the present value of incremental cost) in equation (8) is zero.

$$ROE_L=\frac{ROE_U-aL}{(1-L)} \tag{9}$$

where

$$a=\left|T*\frac{(1+r_{REV})}{(1+r_F)}*r_F+(1-T)*r_F\right|$$

Table 1 compares the cost of equity and WACC of this model with the MM[6] values for hypothetical data. In this table it has been assumed that the risk-free rate is 5%, the expected return on market 11.78%, and that the systematic risk of output (and hence that of the cost and revenue streams) is 0.15, and the income tax rate is 40%.

{TABLE 1 ABOUT HERE}

The cost of equity and WACC, at any leverage level, will always be higher with this model since the firm faces a tariff reduction (and consequent value diminution) when debt is substituted for equity. Unlike the MM model where the debt tax shield adds to shareholder wealth, here the debt tax shield effectively accrues to the consumer. This can be seen formally by comparing the sensitivities of the model ROE_L and the Modigliani-Miller ρ_L, with respect to leverage. If the cost of unlevered equity of the model and MM are equal, a comparison of these two derivatives will show that, cost of equity of the model will always exceed the MM cost of equity, for a leverage L.

$$\partial ROE_L / \partial L = \frac{(ROE_U + a)}{(1-L)^2}$$

and

$$\partial \rho_L / \partial L = \frac{(\rho_u - r_f) * |(1-T)|}{(1-L)^2}$$

Table 2 shows the balance sheets at time '0' of the all-equity firm and the firm with a leverage L= 0.30.[7] With leverage, the expected revenue from consumers declines, and the loss of value to consumers exactly offsets gain from the interest tax shield. The corresponding income statements (also showing free cash flows) are in table 3. These two tables assume (in addition to the assumptions in table 1) that the initial investment is 100 and the expected annual cost is 260.62.

{TABLE 2 AROUND HERE}

{TABLE 3 AROUND HERE}

[6] $\rho_L = (\rho_U - r_F)*(1-t)*L/(1-L)$, where I use ρ to distinguish the MM equation from the model here.

[7] The balance sheet assumes that the depreciation tax shield is discounted at the risk-free rate. A standard text (Brealey and Myers 2003, 546-47) prescription is to discount at the *after-tax* risk-free rate. Such a rate assumes that depreciation tax shields support additional debt. In this paper the debt level is exogenous, and the risk-free rate is appropriate for discounting.

3. THE COST OF EQUITY GIVEN A GENERAL PRICE-ELASTIC DEMAND

I next formulate the cost of equity-leverage relationship for a downward sloping demand with constant price elasticity[8] η (modifying assumption A8 in Section 2).

Suppose, as before, that the regulator fixes a tariff for an all-equity regulated firm and specifies an ROE_U. Substitution of equity by debt will imply a lower tariff. Output, and therefore, expected cost will all increase.

Let the all-equity firm operate with unit tariff P_U and output Q_U, and the levered firm with unit tariff P_L and output Q_L. Equation 9 continues to specify the cost of equity-leverage relationship (this follows by substituting for expected revenue and cost[9] in equations 6, 7, and 8). The key to this relationship continuing to hold is that the discount rates of the revenue and cost streams are equal.

With this levered return on equity, the unit tariff P_L for leverage L can be calculated from equation 7.[10]

A numerical illustration is in tables 4 to 6 (with the same assumptions as in tables 1-3 and for an assumed elasticity of -2). The unlevered firm is assumed to have a unit tariff of 2.8394 and output of 130.3112 (I will address the derivation of these in Section 4). If the leverage is 0.30, unit tariff will be 2.8294 and output 131.2244. With elastic demand, the cost of levered equity in table 4 is identical to that in table 1; however, output increases with leverage. The balance sheets of the levered and unlevered firm are in table 5. With price-elastic demand, the values of both revenues and variable cost of the levered firm change so as to offset the interest tax shield. The corresponding income statements (and free cash flows) are in table 6. Given leverage, the expected cost and expected revenues (tables 3 and 6) are higher for the firm with price-elastic demand compared to the firm with inelastic demand. These differences are reflected in the corresponding balance sheets (tables 2 and 4).

4. THE UNLEVERED COST OF EQUITY REVISITED

I assume the demand function

$$Q(s) = l(s) * P^{\eta}$$

The location parameter l(s) motivates output uncertainty (O'Brien 2005). I assume that it has an expected value E(l) and systematic risk β_l. The betas of output, cost and revenues will equal β_l. In this model the elasticity will be greater than unity. There is one important difference from the O'Brien approach—the unit-tariff here is state-independent.

[8] This in effect makes demand elasticity the same in all states of the world (see Conine 1983).
[9] $E(REV_U)=P_U*Q_U$, $E(REV_L)= P_L*Q_L$, $E(COST_U)=c*Q_U$, and $E(COST_L)=c*Q_L$
[10] Given $(Q_L-Q_U)=(P_L-P_U)*\eta*Q_U/P_U$, a quadratic equation for P_L can be obtained.

The regulator first determines the systematic risk of unlevered equity using equation 4. The regulator then substitutes in equation 1 to obtain equation 10.

$$I * P_U^{\eta+1} = c * I * P_U^{\eta} + I * \left(1 + \frac{ROE_U}{(1+T)}\right) \qquad (10)$$

Solving this for P_U provides the required tariff for the unlevered firm.

An illustration of the unlevered cost of capital computation is provided. Consider a three state world with state probabilities and market prices as in the first two rows of table 7 (panel A). As in Booth (1982) I assume a representative individual with quadratic utility. This information yields a risk free rate of 5.00% and an expected return on the market of 11.78%.

The regulated firm makes an initial investment of 100 at time '0' financed completely by equity, and operates for 1 year. There is no salvage value and deprecation is 100. The income tax rate is 40%.

The firm has a demand function:

$$Q(s) = l(s) * P^{-2}$$

where l(s) is assumed to take values of 1181.87, 1000.00 and 970.00 in the three states, respectively.

These state-contingent values of the location parameter result in the systematic risk of output (and of revenue—given a state-independent unit tariff) being 0.15.

The firm has a cost function:

$$COST = 2 * Q(\mathrm{s})$$

The systematic risk of cost will be equal to that of output.

Given the elasticity of –2, using equation 10 will two yield possible tariffs (2.839 and 6.765). I assume that the regulator chooses the lower tariff. Panel C shows the corresponding state-contingent output. Panel D shows the income statement and the cash flows to equity holders. Note that this output was used as the starting point in Section 3.

With elasticities other than –2, the number of real positive tariffs may be greater than two. It is assumed that the regulator will select the lowest positive values

6. CONCLUSION

In the classic Modigliani-Miller world with corporate tax, the present value of interest tax-shields enhances the value of the unlevered firm. A one-year project that has zero net present value to an all-equity firm, will have positive net present values if the firm uses leverage.

In the case of a regulated firm, the regulator may set the return on equity (and hence the tariff) to maintain "fairness" i.e. ensure that the allowed return on equity do not result in equity investors of a regulated firm making present value gains. This would imply the use of equation (7) to set the return on equity (and the unit tariff) appropriate to leverage. Embedded in this equation is the principle of "fairness". Also embedded is the insight that when leverage increases, the return on equity has to incorporate not only the tax-shield of debt, but also possible changes in the present values of both consumers and input-suppliers.

Equation 7 assumes that there is a single source of systematic risk. The equation also assumes a single-period. With multiple periods the regulator would need to rework the rate of return on equity at the start of each period using the same equation (if one can finesse issues of regulatory lag).

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Table 1: Cost of Equity and WACC: Model Vs. Modigliani-Miller

Leverage D/I	Model		Modigliani-Miller	
	ROE_L	WACC	ρ_L	WACC
0.0	5.63%	5.63%	5.63%	5.63%
0.1	5.70%	5.43%	5.67%	5.40%
0.2	5.78%	5.23%	5.72%	5.18%
0.3	5.89%	5.02%	5.79%	4.95%
0.4	6.04%	4.82%	5.88%	4.73%
0.5	6.24%	4.62%	6.01%	4.50%
0.6	6.54%	4.42%	6.20%	4.28%
0.7	7.05%	4.22%	6.51%	4.05%
0.8	8.07%	4.01%	7.14%	3.83%
0.9	11.12%	3.81%	9.03%	3.60%

The model assumes r_F =5.00%, r_M=11.78%, systematic risks β_{REV}= β_{COST}= β_{OUTPUT}=0.15, and corporate tax rate T=40%

Table 2: Inelastic Demand-Balance Sheets of the Unlevered and Levered Firm at Time 0

Item	L=0	L=0.30	ΔValue
PV Revenue (After-tax)	209.40	208.83	-0.57
PV Cost (After-tax)	-147.50	-147.50	0.00
PV Interest Tax-shield	0.00	0.57	0.57
PV Depreciation Tax-shield	38.10	38.10	0.00
TOTAL ASSETS	100.00	100.00	0.00
Debt	0.00	30.00	30.00
Equity	100.00	70.00	-30.00
TOTAL LIABILITIES & EQUITY	100.00	100.00	0.00

Assumptions as in table 1 together with investment I =depreciation DEPR= 100, and expected cost E(COST)=260.62.

Table 3: Inelastic Demand-Income Statements of the Unlevered and Levered Firm

Item	**L=0**	**L=0.30**
Revenue	370.00	369.00
Cost	260.62	260.62
Contribution	*109.38*	*108.37*
Depreciation	100.00	100.00
Earnings before Interest and Tax	*9.38*	*8.37*
Interest	0.00	1.50
Profit Before Tax	*9.38*	*6.87*
Tax	3.75	2.75
Profit After Tax	5.63	4.12
Flow-to-equity	105.63	74.12
Free Cash Flow [WACC]	105.63	105.02

Assumptions as in tables 1 and 2.

Table 4: Cost of Equity and Leverage with Price-Elastic Demand

Leverage D/I	ROE_L	Unit-tariff	Output
0.0	5.63%	2.8394	130.31
0.1	5.70%	2.8360	130.62
0.2	5.78%	2.8327	130.92
0.3	5.89%	2.8294	131.22
0.4	6.04%	2.8262	131.52
0.5	6.24%	2.8230	131.82
0.6	6.54%	2.8198	132.11
0.7	7.05%	2.8167	132.40
0.8	8.07%	2.8136	132.68
0.9	11.12%	2.8105	132.97

The model assumes r_F =5.00%, r_M=11.78%, systematic risks β_{REV}= β_{COST}= β_{OUTPUT}=0.15, corporate tax rate of 40%, investment I = depreciation DEPR= 100, expected cost of the unlevered firm $E(COST_U)$=260.62, and price-elasticity of demand=-2.

Table 5: Price-Elastic Demand-Balance Sheets of the Unlevered and Levered Firm

Item	**L=0**	**L=0.30**	**ΔValue**
PV Revenue (After-tax)	209.40	209.87	0.46
PV Cost (After-tax)	-147.50	-148.53	-1.03
PV Interest Tax-shield	0.00	0.57	0.57
PV Depreciation Tax-shield	38.10	38.10	0.00
TOTAL ASSETS	100.00	100.00	0.00
Debt	0	30.00	30.00
Equity	100.00	70.00	-30.00
TOTAL LIABILITIES & EQUITY	100.00	100.00	0.00

Assumptions as in table 4.

Table 6: Price-Elastic Demand-Income Statements and Free Cash Flows of the Unlevered and Levered Firm

Item	**L=0**	**L=0.30**
Output	130.3112	131.2244
Unit tariff	2.8394	2.8294
Unit cost	2.0000	2.0000
Revenue	370.00	370.82
Cost	260.62	262.45
Contribution	*109.38*	*108.37*
Depreciation	100.00	100.00
Earnings before Interest and Tax	*9.38*	*8.37*
Interest	0.00	1.50
Profit Before Tax	*9.38*	*6.87*
Tax	3.75	2.75
Profit After Tax	*5.63*	*4.12*
Free Cash Flow Equity	105.63	74.12
Free Cash Flow [WACC]	105.63	105.02

Assumptions as in tables 5 and 6

Table 7: Systematic Risk of the Unlevered Firm with Single Driver of Risk

	State 1	State 2	State 3	Expected Payoff	Value	RADR	Covariance	Beta
Panel A: State Prices								
State Probability π(s)	0.33	0.33	0.33					
Market M(s)	1600	1400	492.97	1164.32	1041.64	11.78%	232026.01	1.00
State Prices P(s)	0.2754	0.2947	0.3823					
$1+r_F$	1.05000							
Panel B: Firm Inputs								
Location parameter l(s)	1181.77	1000.00	970	1050.59	990.97	6.02%	33110.61	0.15
Elasticity	-2.00	-2.00	-2.00	-2.00				
Unit cost c	2.00	2.00	2.00	2.00				
Panel C: Firm Output-Tariff Decision								
P	2.84	2.84	2.84	2.84	2.70	5.00%	0.00	0.00
Q	146.58	124.04	120.32	130.31	122.92	6.02%	4106.92	0.15
Panel D: Firm Income Statement for Year 1								
Revenues P*Q	416.20	352.19	341.62	370.00	349.01	6.02%	11661.16	0.15
Cost c*Q	293.16	248.07	240.63	260.62	245.83	6.02%	8213.84	0.15
Depreciation	100.00	100.00	100.00	100.00				
Interest	0.00	0.00	0.00	0.00				
Profit Before Tax	23.04	4.12	0.99	9.38	7.94			
Tax	9.22	1.65	0.40	3.75	3.17			
Profit After Tax	13.82	2.47	0.60	5.63	4.76			
Cash-Flow to Equity	113.82	102.47	100.60	105.63	100.00	5.63%	2068.39	0.0929

1. State contingent prices derived from: $P(s) = \pi(s) / P_0 [1-2*vM(s)]/[1-2vM_0]$ where π(s) are the state probabilities and M(s) are market endowments. Assuming v=0.00011, P_0 =1 and E_0=1000 yields P(1) = 0.2754, P(2) = 0.2947 and P(3) = 0.3823
2. Risk-free Rate $R_F = 1/\Sigma P(s) = 5\%$
3. Expected Payoff = Σ π(s)*State Payoffs
4. Value = Σ P(s)*State Payoffs
5. Expected Market Return = Expected Market Payoff/Market Value -1= 11.78%
6. Covariance = Covariance between Market and Security (revenues, costs, profits and so on each constitute a security).
7. Beta = Covariance/Variance of Market*Value of Market/Value of Security

See Booth (1982) for the formulae