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Supply Chain Contracts with Capacity Investment Decision: Two-way Penalties for Coordination

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1. Abstract

Supply chain contracting has been discussed usually under the two compliance regimes, forced and voluntary. Various contracts show different coordination characteristics in the two regimes. However, in practice, the enforcement of quantities in case of forced compliance is always an issue of concern. We propose a *price compliance* regime for contract where the penalties are enforceable on both parties, which are price for non compliance on quantities. In this paper, we attempt to model a contract where supplier needs to build capacity before demand is realized under the proposed *price compliance* regime. A need for investment in capacity can be of the form of new capacity installation or capacity enhancement or updating, and is prevalent in practice, especially in industries which witness shorter product life cycles, high rate of new product launches, and in high-tech industry. Since under-investment in capacity by supplier is a major concern for manufacturer, we model contract with supply chain capacity and cost to build the capacity are included in the model and analyze how the manufacturer can influence the capacity decision of supplier with the given contract. We analyze the impact of various penalty parameters on vendor's capacity decision, supply chain efficiency and relative allocation of supply chain profit across partners. Further, we consider a special case of uniformly distributed demand and find analytical closed form conditions for a sub-set of coordination conditions. We also consider a special case where buyer and supplier arrive at consensus on capacity related decision and capacity is verifiable by buyer.

Key words: Supply chain coordination, contract, capacity reservation contracts, penalty.

2. Introduction

In a supplier-manufacturer relation, the manufacturer buys parts or products (or services) from supplier and after value adding, sells it to the market. The supplier needs to build a capacity to supply the order quantity, and if capacity installation has a lead time, the decision on capacity is required to be taken much prior to receiving the firm orders from manufacturer. With the costs involved in building the capacity, 'how much capacity to build' is a difficult decision for supplier since she has to take the risk of over-capacity. When the cost of building capacity is high, supplier would expect some purchase commitment from manufacturer before investing in capacity building to partially cover the downside risk. On the other hand, if the supplier happens to be a sole supplier for that product, higher capacity at supplier end would make the manufacturer better-off and hence he would try to induce the supplier for building higher capacity, and would expect a capacity investment up to a certain level to limit the risk of loosing sales. With all these issues involved, the purchase agreement or contracts need to be designed in such a manner that the two players involved can ensure their returns in proportion to the risk born by them and bargaining power they enjoy between the two.

Recently, IBM has launched its 'on-demand' services, where IBM, the supplier, would build the capacity and user of the services will pay only for the resources used on an as-needed basis rather than the capacity reserved with assurance to service to the future growth of the clients. This agreement offers a great potential for the buyers to reduce the capacity costs substantially, and on the other hand, supplier can leverage on economies of scales. The supplier would install required resources in place with an assurance to buyers to cater to their future expansion in demand. However, as the demand grows, the buyer would be at disadvantage if supplier lands up in a situation of capacity scarcity and assigns resources based on its own priorities. Therefore, the buyers need to negotiate for the service levels in the contracts and supplier needs to know the planned demand from the buyer, which is a forecast of demand in the planning horizon.

The industries which witness short life cycles of products, high rate of product launches and evolving technologies require regular investments in capacity, in form of new capacity installation or capacity enhancement or updating, which has a long lead time and at the same time it is costly. In such environment, supplier-manufacturer are often involved in a contract which require to make a decision on capacity investment and sharing of risks. Jin and Wu [9], in the context of high tech industry like semiconductor, telecommunication and optoelectronics, discuss the capacity reservation contracts for coordination. Similarly, in a recent study, Durango-Cohen and Yano [7] study the application specific integrated circuits industry where buyers book the capacity at suppliers end, and design a forecast-commitment contract and show the optimal commitment strategy for supplier.

Motivation for our work also comes from the similar contexts where capacity is costly, has a lead time, and supplier needs to take a decision on how much capacity to build before demand is realized. We try to develop a contract which can be implemented to achieve the supply chain coordination.

A supply chain is coordinated when the decentralized system allows the individual business entities to perform for their individual optimization and at the same time their actions are aligned so as to maximize the over-all supply chain performance, at a level that is as good as centralized one. The literature on coordination and contracting broadly tries to address two issues. First, in a given context, can the supply chain be coordinated? If not, what is the efficiency at which supply chain is operating, where efficiency is the ratio of uncoordinated profits to coordinated profits of entire supply chain. Second, how to create incentives for supply chain partners to implement the actions, which coordinate the supply chain. What values of contract parameters will induce the coordination behavior of the two rational players? Further, would the contract have voluntary compliance, or forced compliance?

Researchers have discussed the two compliance regimes, forced and voluntary compliance under various contracts. Various contracts show different coordination characteristics in the two regimes. The forced compliance assumes that in case of failing to adhere to the contracted quantities, the legal action can be initiated to force adherence to the contracted quantities. This implies that the penalty or cost implications for not adhering to contracted quantity are infinitely high. On the other hand, the voluntary compliance assumes that being rational business entity, firms would try to optimize their profits and would adhere to contracted quantities as far as it is optimal for them. Hence, the penalty for not adhering to contracted quantities is in the form of sub-optimal performance.

However, in practice, the enforcement of quantities in case of forced compliance is always an issue of concern. The enforcement of quantities can never be guaranteed. Though, a specific contract may coordinate in forced compliance regime, if the quantities cannot be ensured, it is of no consequence to the coordination. On the other hand, most of the contracts are found not coordinating in the voluntary compliance. Therefore, in this paper, we argue that though the quantities cannot be forced, all the costs can be enforced legally and propose a *price compliance* regime for contract where the penalties are enforceable on both parties. These penalties represent the price for not complying to the quantities.

In this paper, we attempt to model a single supplier - single manufacturer supply chain where the supplier needs to build capacity before producing and manufacturer reserves the capacity for his supplies. The supply chain is modeled with capacity as a parameter of contract under *price compliance* regime. Further impact of various penalty parameters on vendor's capacity decision, supply chain efficiency and relative allocation of supply chain profit across partners are analyzed. We also analyze a special case where buyer and supplier arrive at consensus on capacity related decision and capacity is verifiable by buyer.

A brief literature review follows in next section, which is followed by the model description in section-4. The generic model is discussed in section-4.1, followed by special case-1 where demand is uniformly distributed. The discussion on the special case-1 is taken up in the same section, followed by an illustration. Section-4.3 presents the special case-2 where capacity is visible and manufacturer and supplier jointly decide on capacity. Last section, section-5 concludes the paper.

3. Literature Review

In the last decade, plenty of work has been done in the area of supply chain contracting. Though, the literature has many studies with various variables included in the study like price, selling effort etc., the literature review here is restricted to two-echelon supply chain models where market price is exogenously given and demand is stochastic, independent of market price, effort level, or any other variable.

Tsay et al [17] pointed out that supply-chain contracts are used to ensure that the supply-chain profits in decentralized decisions are as close to that in decentralized decisions as possible, if not equal, so, they have system wide performance improvement objective. Further, some contracts focus on splitting of profits between the two parties, and sharing the risk arising from various uncertainties.

In one of the early studies, Pasternack [13] showed that it is possible for a manufacturer to set a pricing and return policy that will ensure channel coordination. They showed that no return policy as well as full return is sub-optimal, whereas, unlimited quantity partial return is optimal, and is independent of retailers demand function. Optimal pair of selling price and return price depends solely on manufacturing cost, retail sales price, salvage value, and goodwill price. Lariviere [10] discussed price only contracts and shows it does not coordinate the supply chain, because double marginalization takes place, whereas buy-back contracts do coordinate. They discussed quantity flexibility contracts and show that a continuum of coordination condition exists, but provides limited flexibility, which is a function of distribution parameters, hence author favors buy-back contracts.

Cachon [3] applied various contracts to one-supplier, one-retailer models with single period stochastic demand. They show that buyback and revenue sharing contracts coordinate under voluntary compliance with a flexibility in profit distribution, whereas wholesale price, quantity flexibility, sales rebate, quantity discount contracts do not ensure coordination under voluntary compliance. Tsay and Lovejoy [16] showed the coordinating mechanism with quantity flexibility contracts in rolling horizon planning where lower bound act as minimum commitment and upper bound prevents buyer from overstating its demand. Li and Liu [11] also discussed the quantity discount policy in a multi-period horizon. Bassok and Anupindi [2] considered a total minimum quantity commitment contract in a multi-period setting and identified the structure of the optimal purchasing policy given the commitment. Taylor [14] shows that a target rebate achieves channel coordination with both parties better off. Coordination cannot be achieved by a linear rebate, which is implementable. However, when the demand depends on sales efforts, contracts like linear rebate and return, or target rebate alone cannot achieve coordination, but a properly designed target rebate and return achieves channel coordination and win-win situation.

Cachon and Lariviere [6] modeled the supply chain where retailer has more information about the demand, and the capacity at supplier is always a bottleneck, and showed that retailer has incentives to inflate the demand forecasts. Cachon and Lariviere [5] studied the same model in a game-theoretic set-up with options, and discussed the contracts under both forced and voluntary compliance. They showed that the compliance regime affects the outcomes considerably. In full information scenario, in forced compliance regime, manufacturer dictates the terms and offers contract which offer supplier his minimal acceptable profit. Barnes-Schuster et al [1] also studied the role of options in supply chain coordination in the two compliance regimes in a two period model. Wang and Tsao [18] introduced a contract with bi-directional options that can be exercised as both call options and put options in a predetermined manner, and determine the buyer's optimal policies. Cachon [4] on the contrary claims that the literature has exaggerated the importance of implementing coordinating contracts, and if firms follow advance purchase contract, with two wholesale prices, supply chain coordination is possible with arbitrary allocation of profits.

Relaxing the assumption of no capacity constraints of supplier, Erkoc and Wu [8] study the capacity reservation contracts with exogenous wholesale price and selling price in fully deductible reservation fee setup and find that channel coordination is achievable only under very restrictive conditions. Jin and Wu [9] also discussed the capacity reservation contract in high tech industries like semiconductor, telecommunication etc, with deductible reservation and show that coordination conditions do exist. When both supplier and manufacturer invest in capacity, Tomlin [15] introduced alternative mechanism of sharing the gain of high demand and showed that a nonlinear contract coordinates the supply chain. The option model of Cachon and Lariviere [5] also had the cost of building capacity as a model variable. A detailed review of capacity management literature is presented by Wu et al [19].

Durango-Cohen and Yano [7] discuss forecast commitment contract where supplier builds capacity based on manufacturer's forecast, and analyze it from supplier's point of view to derive optimal quantities. However, this type of contract still has a bias towards buyer, and operates in buyer dominant context. Further, it does not diminish the buyers' tendency of inflating the forecasts. Ozer and Wei [12] discuss two different contracts, nonlinear capacity reservation contract and advance purchase contract, and show that coordination is possible in even asymmetric demand information.

In the proposed model, we attempt to present a comprehensive model in capacity procurement setting to help supply chain partners in designing appropriate contract parameters to coordinate or enhance supply chain performance, which the current literature does not seem to address satisfactorily.

4. The Model

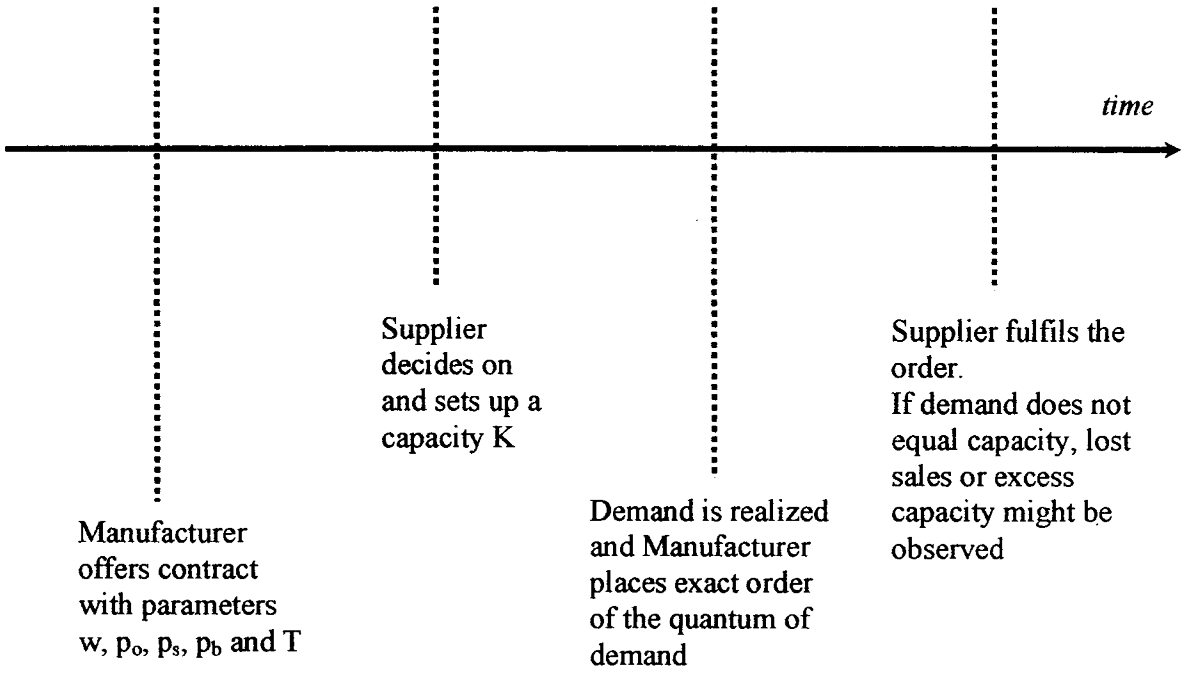
The modeled supply chain comprises of one manufacturer and one supplier in a single period setting where this supplier is the sole supplier for the particular product to manufacturer. The supplied product is worked upon by manufacturer and value added finished product is sold to the market. Before the supplier supplies the product, she needs to build a capacity, since capacity building has a lead time, and capacity cannot be build on incremental basis. Demand is realized after the capacity is built, and final production takes place only after the demand is realized. The product is perishable in nature and hence cannot be stored in any form, WIP or FG inventory. Hence, the demand in excess of capacity would result in lost sales, whereas if demand is less than the capacity, there will be under-utilization of capacity. The market demand follows a continuous distribution F(x) with density f(x), differentiable for $x \geq 0$.

The manufacturer earns revenue (r) by selling the product in market. He pays a wholesale price (w) to supplier for each unit product purchased, where $r > w$. The marginal cost for building capacity and marginal cost of production are c_k and c_p respectively for supplier, where $w > c_k + c_p$. We assume, in the model, that the manufacturer does not incur any cost on capacity building or on procurement.

Cachon [3] assume that the wholesale price operates with forced compliance. We extend this idea to introduce the *price compliance* regime, where all prices can be enforced whereas quantities cannot be enforced. We introduce two-way penalties in the contract which can be enforced, and operate under force compliance, and are representative of prices the parties pay for non-compliance on quantities. The supplier pays a penalty (p_s) for each unit undersupplied whereas manufacturer pays a penalty (p_o) for each unit underutilized of reserved capacity, which we term as target capacity (T). We introduce another parameter, which is bonus (p_b) to supplier for supplying above the pre-decided target capacity. This target capacity (T) is used as a reference to calculate the two-way penalty as well as bonus.

The sequence of event is that manufacturer, being the first mover, offers a contract (with parameters w, p_o, p_s, p_b and T), which is "accept or reject" type of contract. If supplier accepts the contract, she decides on and installs the capacity (K). Now demand is realized and manufacturer places exact order of the quantum of demand. The supplier fulfils the order and if demand does not equal capacity, lost sales or excess capacity might be observed.

Figure 1 : Sequence of events



The decision problem for supplier is to install a capacity (K) that maximizes her profits, given the contract parameters w, po, ps, pb and T. Manufacturer's decision problem is to set the contract parameters which will maximize his profits, given supplier's response to the contract.

4.1 Generic Mode

Let $S(K)$ represent the Sales for a given capacity (K), it would be given as:

$$S(K) = K.\overline{F}(K) + \int_0^K x.f(x).dx$$

$$S(K) = K - \int_0^K F(x).dx$$

where, $\overline{F}(K) = 1 - F(K)$

Short Supply Quantity (S_s)

The supplier will compensate the manufacturer by paying Short Supply Penalty (p_s) on this quantity. This Short Supply Quantity is defined as the shortfall in the supplied quantity with respect to the Target (T) or actual demand (D), whichever is lesser.

$$S_s = \left(Min(T,D) - S(K)\right)^+$$

where, function $(y)^+$ is defined as: $(y)^+ = \max(y,0)$

Short Order Quantity (S_o)

The manufacturer will compensate the supplier by paying Short Order Penalty (p_o) on this quantity. Manufacturer has offered the contract specifying a Target, which in a sense is like sharing the demand information, and he expects the supplier to set her capacity to meet at least this much demand. Since supplier incurs a cost in building this capacity, the manufacturer compensates her in case the actual demand is below the Target.. Manufacturer will compensate her for the shortfall in the ordered quantity with respect to the Target (T).

However, the capacity at supplier's end is not visible to manufacturer, and cannot be verified by supplier. In such case, the manufacturer would compensate for short orders only when his ordered quantity (up to Target level) is fulfilled, since the supplied quantity is the only parameter for manufacturer to sense the installed capacity. Hence, if it is revealed that supplier has not built a capacity up to Target, manufacturer will not compensate for short order.

$$S_o = \int_0^K (T - D)^+ . f(x).dx$$

This short order penalty is essential for a feasible system. If manufacturer had to offers a contract with $p_o = 0$, his profit would be increasing in T and hence his optimal target setting would be at highest value possible. As a result, the supplier would not accept such contract, and even if she accepts, her expected loss in the form of short supply penalty will no more be a function of T, and it will just depends on the distribution function of demand. Since the demand function is known to the supplier a priori, target setting will not influence his capacity decision in any way.

Supply Quantity with Bonus (S_b)

The manufacturer rewards the supplier whenever she supplies for a demand above the Target level by paying Excess Supply Bonus (p_b) on this quantity. This, in a sense, represents a penalty on manufacturer for under-estimating the Target. Further, the supplier might be supplying the same product to some other markets also, and hence might install a capacity higher that the Target. This bonus represents the incentive for him to supply quantity to manufacturer, rather than other markets, whenever demand exceeds the Target.

$$S_b = \left(S(K) - T\right)^+$$

Let the Transfer Payment from manufacturer to supplier be (T_r), which is given by following expression:

$$T_r = w.S(K) + p_o \cdot S_o - p_s \cdot S_s + p_b \cdot S_b$$

Supplier's profit (π_s) and Manufacturer's profit (π_m) will be:

$$\pi_s = T_r - c_p.S(K) - c_k.K$$

$$\pi_m = r.S(K) - T_r$$

The Supply Chain Profit (Π) will be:

$$\Pi = (r - c_p)S(K) - c_k.K$$

If there is a centralized decision making for the supply chain, optimal capacity for integrated supply chain (K^*) would be given by:

$$\overline{F}(K^*) = \frac{c_k}{r - c_p} \quad ......(1)$$

In absence of any penalty structure (i.e., $p_s = p_o = p_b = 0$), the optimal capacity for supplier (K_s^*) can be determined by solving his individual newsvendor problem, and is given by:

$$\overline{F}(K_s^*) = \frac{c_k}{w - c_p}$$

It is straight forward to note that when w < r, $K_s^* < K^*$, which means the supplier would build lesser capacity than required for integrated chain's optimal performance. Hence, supply chain coordination cannot be achieved by simple wholesale price contract. Now we introduce the three penalties and Target capacity, which are set by the manufacturer, in the contract and see how the inclusion of these parameters enables for supply chain coordination.

With the given contract with penalties and target capacity, supplier decides on an optimum capacity to be build. We consider two scenarios separately, scenario-1 where supplier sets a capacity less than (or equal to) Target, and scenario-2 when he sets a capacity higher than Target since the profit function for supplier differs in the two scenarios. Both these scenarios are discussed separately below. The expressions for Short Supply Quantity, Short Order Quantity, Supply Quantity with Bonus, Transfer Payment, and Supplier's and Manufacturer's profits for the two scenarios are given. Subscripts 1 and 2, wherever used, represent scenario-1 and scenario-2 respectively.

Scenario-1: $K \leq T$

$$S_{s1} = S(T) - S(K)$$

$$S_{o1} = (T - K).F(K) + K - S(K)$$

$$S_{b1} = 0$$

$$T_{r1} = (w - p_o + p_s).S(K) - p_s.S(T) + p_o.(T - K).F(K) + p_o.K$$

$$\pi_{s1} = (w - p_o + p_s - c_p).S(K) - p_s.S(T) + p_o.(T - K).F(K) + (p_o - c_k)K$$

$$\pi_{m1} = (r - w + p_o - p_s).S(K) + p_s.S(T) - p_o.(T - K).F(K) - p_o.K$$

Supplier's marginal profit is given by:

$$\frac{\partial \pi_{s1}}{\partial K} = (w + p_s - c_p).\bar{F}(K)\left[1 - \frac{p_o}{(w + p_s - c_p)}.(K - T).\frac{f(K)}{\bar{F}(K)}\right] - c_k$$

From the above expression, it is evident that $\frac{\partial \pi_{s1}}{\partial K}$ decreases in K if demand distribution has increasing failure rate (IFR). Hence, with IFR demand, π_{s1} is unimodal, and there is a unique capacity K^*_{s1} where supplier's profit maximizes, i.e., $\pi_{s1} = \pi^*_{s1}$, and can be found by solving the following expression for K.

$$(w+p_s-c_p).\overline{F}(K)\left[1-\frac{p_o}{(w+p_s-c_p)}.(K-T).\frac{f(K)}{\overline{F}(K)}\right]-c_k=0 \quad \text{......(2)}$$

Scenario-2: K > T

$$S_{s2}=0$$

$$S_{o2}=T-S(T)$$

$$S_{b2}=S(K)-S(T)$$

$$T_{r2}=(w+p_b).S(K)-(p_o+p_b).S(T)+p_o.T$$

$$\pi_{s2}=(w+p_b-c_p).S(K)-(p_o+p_b).S(T)+p_o.T-c_k.K$$

$$\pi_{m2}=(r-w-p_b).S(K)+(p_o+p_b).S(T)-p_o.T$$

Supplier's marginal profit is given by:

$$\frac{\partial \pi_{s2}}{\partial K}=(w+p_b-c_p).\overline{F}(K)-c_k$$

It is straight forward that $\frac{\partial \pi_{s2}}{\partial K}$ decreases in K, hence π_{s2} is unimodal, and there is a unique capacity K^*_{s2} where supplier's profit is maximized, i.e., $\pi_{s2}=\pi^*_{s2}$, and follows the condition:

$$\overline{F}(K^*_{s2})=\frac{c_k}{w+p_b-c_p} \quad \text{......(3)}$$

Now, considering the two scenarios together, we can say that for the given contract, supplier's rational decision to her problem would be to install a capacity:

$$K_s^* = \{K_{s1}^*, \;\; K_{s2}^*, \;\; T\} ......(4-a)$$

where K_s^* is given by:

$$K_s^* = \begin{cases} K_{s1}^* & if \left(\pi^*_{s1} \geq \pi^*_{s2} \;\&\; K^*_{s1} \leq T\right) \text{ or } \left(\pi^*_{s1} > \pi_{s2}(\mathrm{K}=\mathrm{T}) \;\&\; K^*_{s1} \leq T \;\&\; K^*_{s2} \leq T\right) \\ K_{s2}^* & if \left(\pi^*_{s1} < \pi^*_{s2} \;\&\; K^*_{s2} > T\right) \text{ or } \left(\pi^*_{s2} > \pi_{s1}(\mathrm{K}=\mathrm{T}) \;\&\; K^*_{s1} > T \;\&\; K^*_{s2} > T\right) \\ T & \text{otherwise} \end{cases}$$

$......(4-b)$

In other words, we can say that supplier's response would be to install a capacity equal to any of the three values, K_{s1}^*, K_{s2}^* or T. Proof for the same is presented in Appendix-1.

Since, we have obtained the optimal capacity values for supplier which is dependent on the demand function, in the following sub-section we discuss a special case where demand follows a specific distribution function, a uniform distribution, and show that penalty structure can be designed in a fashion which coordinates the supply chain. An illustration is presented in order to explain the effect of penalties and supplier's response to target setting. Later we discuss another special case, where the capacity is visible to the manufacturer, and supplier and manufacturer jointly decide on capacity..

4.2 Special Case-1

Now we consider a special case where demand follows a continuous uniform distribution, Uniform~(0, a). Hence, $f(K) = \frac{1}{a}$, $F(K) = \frac{K}{a}$, and sales $S(K) = \left(K - \frac{K^2}{2.a}\right)$. Now considering both the scenarios one by one:

Scenario-1: $K \leq T$

Supplier's marginal profit is:

$$\frac{\partial \pi_{s1}}{\partial K} = (w + p_s - c_p - c_k + \frac{p_o}{a}.T) - (w + p_s - c_p + p_o).K$$

which is decreasing in K. The unique capacity K^*_{s1} where supplier maximizes her profit will be given by:

$$K^*_{s1} = \frac{(w + p_s - c_p - c_k)}{(w + p_s - c_p + p_o)}.a + \frac{p_o}{(w + p_s - c_p + p_o)}.T \quad ......(5)$$

Scenario-2: $K > T$

Supplier's marginal profit is:

$$\frac{\partial \pi_{s2}}{\partial K} = (w + p_b - c_p - c_k) - \frac{(w + p_b - c_p)}{a}.K$$

which is decreasing in K. The unique capacity K^*_{s2} where supplier maximizes her profit will be given by:

$$K^*_{s2} = \left(1 - \frac{c_k}{(w + p_b - c_p)}\right).a \quad ......(6)$$

Supply Chain Coordination

For this special case of uniformly distributed demand, the optimal capacity for integrated supply chain (K^*) from equation (1) translates to:

$$K^* = \frac{r - c_p - c_k}{r - c_p}.a \quad ......(7)$$

The supply chain would be coordinated in both the scenarios when optimal capacity of supplier matches with the optimal capacity for integrated supply chain. hence those conditions are obtained by separately equating K_{s1}^* and K_{s2}^* to K^*. Simplification and rearrangement leads us to following conditions which would separately ensure coordination:

Scenario-1: $K \leq T$

$$\left.\begin{aligned} T_1^C &= \left[1+\frac{c_k}{p_o}.\left(\frac{(r-w)-(p_s+p_o)}{(r-c_p)}\right)\right].a \quad ......(8-a)\\ T_1^C &\geq \left[1-\frac{c_k}{w+p_s-c_p}\right].a \quad ......(8-b)\end{aligned}\right\} \quad ......(8)$$

where T^C represents the Target which coordinates the supply chain, and subscript 1 and 2 represent scenario-1 and scenario-2 respectively.

The first condition above (8-a) is obtained by simplifying the condition $K_{s1}^* = K^*$, and the second condition (8-b) is obtained by simplifying $K_{s1}^* \leq T$, which is nothing but the assumption for this scenario. Both these conditions combined, (8) above, represent a sub-set of operating points which coordinate the supply chain. Appropriately chosen contract parameters would meet the condition (8) above and result in supply chain coordination.

Scenario-2: $K > T$

$$\left.\begin{aligned} p_b &= r-w \quad ......(9-a)\\ T_2^C &< \left[1-\frac{c_k}{w+p_b-c_p}\right].a \quad ......(9-b)\end{aligned}\right\} \quad ......(9)$$

Similar to scenario-1, the two conditions here (9-a) and (9-b) are obtained by the conditions $K_{s2}^* = K^*$, and $K_{s2}^* > T$ respectively. Both these conditions combined, (9) above, represent another sub-set of operating points which coordinate the supply chain and appropriately chosen contract parameters would result in supply chain coordination.

Discussion

As it is evident from (5), the capacity decision of supplier is positively influenced by target setting when $K_s^* = K_{s1}^*$ as per condition (4). Higher target setting would influence the supplier to increase her capacity. This result is quite intuitive as with higher targets, the expected loss in the form of short supply penalty increases and hence she builds a higher capacity. However, an interesting result from (5) follows that the manufacturer cannot influence her capacity decision by setting higher target if he offers a contract with no short order penalty, i.e., $p_o = 0$ (keeping other contract parameters constant). Similar conclusion can be drawn by examining (8-a), where, if $p_o = 0$, no feasible target can coordinate the supply chain, other than the maximum possible target, which would not be accepted by the supplier. Hence, coordination can not be achieved with just short supply penalty and excess supply bonus.

Examining (5) also indicates that short supply penalty p_s has a positive influence on supplier's optimal capacity, which is again quite intuitive, but this influence diminishes when short order penalty p_o increases.

Given that the supplier's capacity exceeds the Target, i.e. contract operates in scenario-2, the only contract parameter that can influence it is the excess supply bonus p_b, as it is evident from (6). The higher bonus for excess supply will induce supplier for setting higher capacity. This explanation again is straight forward as in this scenario, there can be no short supply, so short supply penalty can not influence capacity. When there will be no short supply, whenever demand will fall short of Target, supplier will be compensated, or in other words, reference for calculating short order quantity will always be the Target, so short order penalty will also not influence capacity. Hence only influencing factor is the bonus for excess supply.

Condition (8) represents one sub-set of contract parameters which coordinate the supply chain when $K_s^* = K_{s1}^*$ as per condition (4). Careful evaluation of (8-a) reveals that $(r-w) > (p_s + p_o)$ would result in $T_1^C > a$, which would not be a valid Target for the contract. But $(r-w) \leq (p_s + p_o)$ would result in $T_1^C \leq a$, and hence it would be a valid Target and a valid contract parameter. This implies that whenever $K_s^* = K_{s1}^*$ as per condition (4), a combination of p_s and p_o exists which would coordinate the supply chain.

Condition (9) represents another sub-set of contract parameters which coordinate the supply chain when $K_s^* = K_{s2}^*$ as per condition (4). Coordination condition here is $p_b = r - w$, which means that if manufacturer passes on his complete margin to supplier in the form of bonus for excess supply, this would result in supply chain coordination. When complete profits from sales accrue to supplier, her rational decisions are exactly identical to that of centralized decision maker, and hence the supply chain coordinates.

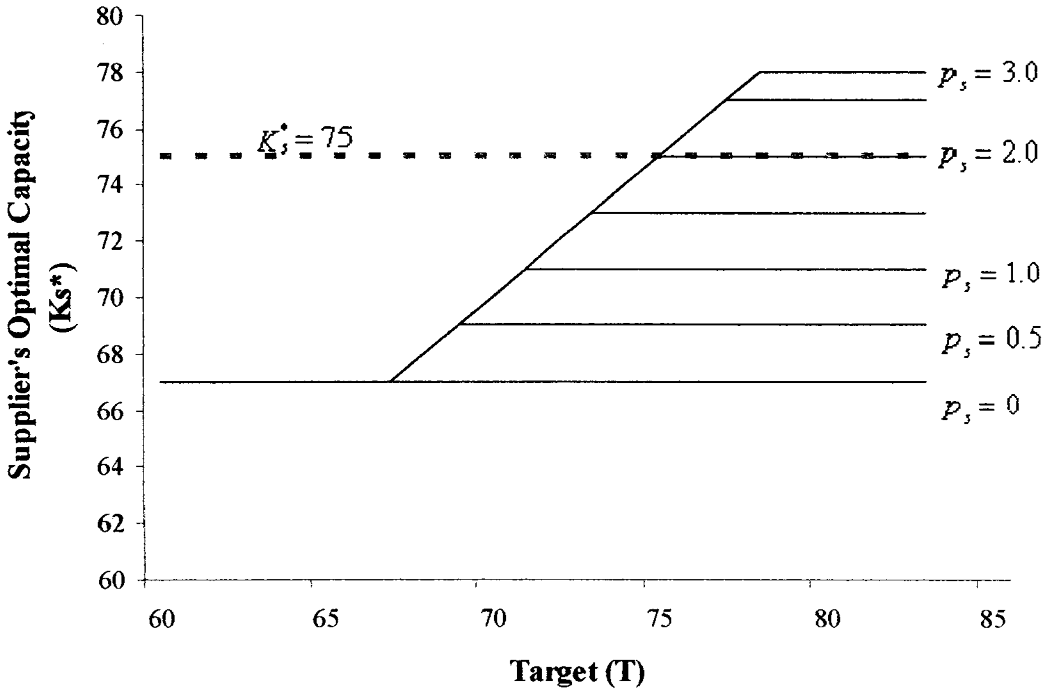
Illustration

For illustration purpose, a set of scenarios were built to demonstrate the effect of proposed contract parameters on supply chain performance and coordination. Demand distribution chosen was Continuous Uniform~(0,100). Selling price (r) and wholesale price (w) were set to 10 and 8 respectively and marginal cost for building capacity, c_k =2 and marginal cost of production, c_p =2 for supplier. The sequence of events here was same as conceptualized in model, i.e., the manufacturer offers a contract with parameters w, p_s, p_o, p_b and T and with those parameters, supplier decides on optimal capacity. The three penalties were increased in steps to create various combinations of contract parameters, and Target was calculated as per condition (8) and (9), whichever applicable. Supplier's decision of capacity was made based on condition (4), (5) and (6). supplier's optimal capacity thus calculated subsequently determines supply chain performance.

First, to understand the effect of each penalty on capacity decision, each one of them is varied individually and its influence on supplier's capacity decision is observed.

As found in the analytical expression, the penalty for short supply (p_s) influences supplier's decision of capacity installation (refer figure-2). With increasing p_s, supplier's optimal capacity increases. Target set above a threshold level can induce her to increase her capacity up to a certain level. However, this increase in capacity reduces with increase in P_s. Below this threshold level, supplier's optimal capacity is more than the target set, hence the contract is operating in scenario-2 (K>T). In this scenario, as is evident from condition (9-a), p_s does not influence supplier's capacity decision, and hence all the curves for different values of p_s coincide. The capacity increases linearly with target in the middle portion of the graph. This represents the condition when contract operates in neither of the scenarios, and as per condition (4), supplier sets capacity exactly equal to the target. For the higher targets, contract operates in scenario-1 (K<T), and supplier's optimal capacity increases with p_s.

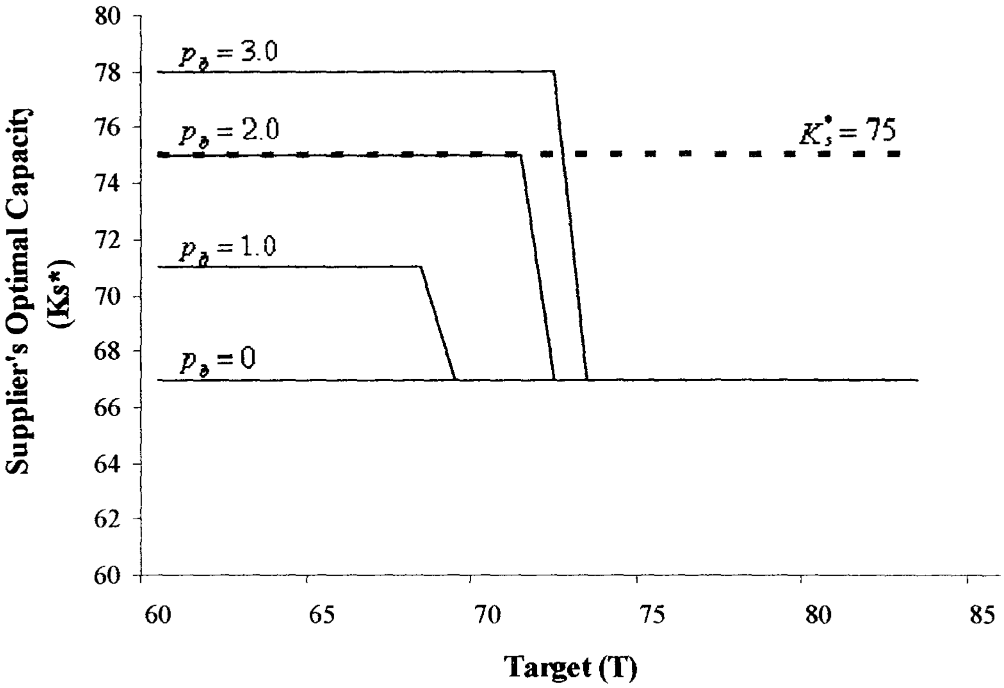
Figure-2 : Effect of p_s on Supplier's optimal capacity decision



$r = 10,\ c_y = 2,\ c_k = 2,\ w = 8,\ p_o = 0,\ p_b = 0$

The excess supply bonus (p_b) influences supplier's decision of capacity in reverse manner (refer figure-3). At lower targets, supplier tends to build higher capacities since his expected profit in the form of bonus is high. But as soon as the target crosses a threshold, the cost of capacity building overweighs this expected profit, and her optimal capacity drops to a lower level. However, incremental drop in capacity reduces with increase in p_b. At lower targets, contract operates in scenario-2 (K>T), and we can straightaway observe that condition (9-a), i.e., $p_b = 10-8 = 2$, coordinates the supply chain. At higher targets, contract operates in scenario-1 ($K \leq T$), so p_b does not influence supplier's capacity decision, and all curves for different values of p_b coincide.

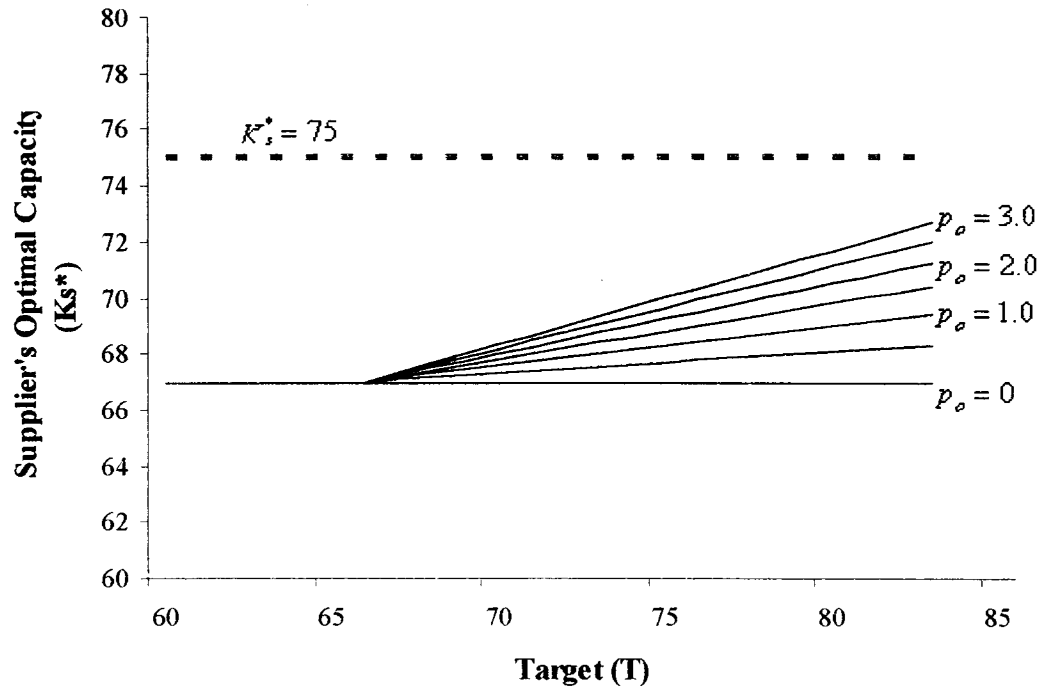
Figure-3 : Effect of p_b on Supplier's optimal capacity decision



$r = 10, \; c_p = 2, \; c_k = 2, \; w = 8, \; p_s = 0, \; p_o = 0$

The penalty for short order (p_o) influences the capacity installation decision positively (refer figure-4). As is evident from condition (5), when p_o is increased, the supplier's optimal capacity increases by a fraction of Target. Same pattern can be observed in the figure. The intuitive explanation for this is that the supplier does not realize any profit in the form of compensation for short order unless he meets the demand (note that manufacturer does not compensate her when short capacity is revealed). Hence, to realize those profits, supplier must build some extra capacity, which would be a fraction of Target based on trade offs between supplier's margin, penalty for short supply and penalty for short order. The p_o starts influencing supplier's capacity decision after a threshold value of T, below which, contract operates in scenario-2 (K>T) and in line with condition (9-a), p_o does not influence supplier's capacity decision, and hence all the curves for different values of p_o coincide up to the threshold.

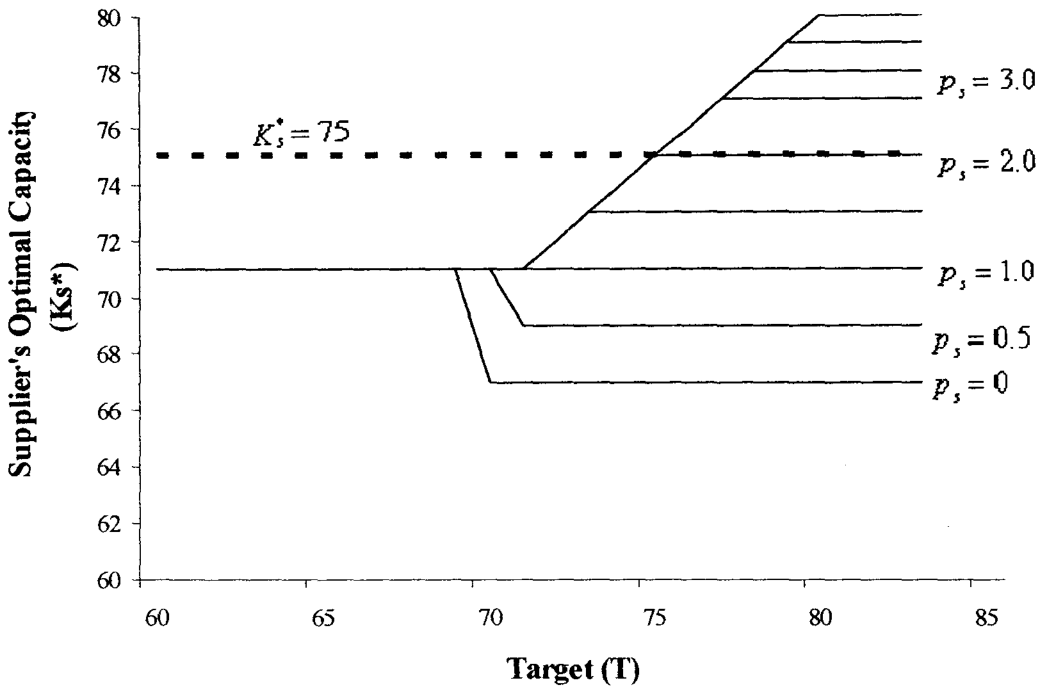
Figure-4 : Effect of p_o on Supplier's optimal capacity decision



$r = 10,\ c_p = 2,\ c_k = 2,\ w = 8,\ p_s = 0,\ p_b = 0$

Though, the influence on supplier's capacity decision behavior of individual penalty in isolation is fairly intuitive and can be followed easily with the help of analytical expressions and illustration above. However, the target setting with two penalty and one bonus terms is not very intuitive. In presence of two penalties, p_o and p_s, the target if set above a threshold level might lead to either of the two, decrease or increase of optimal capacity for supplier (refer figure-5).

Figure-5 : Complexity of Target Setting



$r = 10,\ c_p = 2,\ c_k = 2,\ w = 8,\ p_o = 1.0,\ p_b = 0$

The presence of two penalty and one bonus terms along with the target setting enable the supply chain to approach the coordination. In this illustration, there exists continuum of contract parameters which coordinates the supply chain in scenario-1 (K<T), which is given by condition (8). Detailed illustrated on this is given in Appendix-2.

All these coordinating contracts together provide a wide range of profit allocation and hence flexibility in sharing gains. Those coordinating contracts result in 1.26% improvement in supply chain profits. A list of a few such coordinating contracts under scenario-1 (K≤T) is given in Appendix-3, where the penalties are bound by the limiting condition (12) (mentioned in appendix-2). Details of contract performance are also given in the same annexure.

4.3 Special Case-2

Now we consider the special case where the capacity is visible. Hence, at any point in time, the manufacturer can verify the installed capacity without incurring any cost. In this case, manufacturer and supplier set the target jointly. The manufacturer would enter the contract only when he observes the capacity being set at mutually agreed upon level. In this case, the penalties are calculated with respect to the installed capacity (K) itself, and T is nothing but K. Hence:

$$S_o = (K - D)^{+} \quad = K - S(K)$$

$$S_s = (D - K)^{+} \quad = \mu - S(K)$$

$$S_b = 0$$

(Where μ = mean of demand distribution)

Transfer payment (T_r), Supplier's profit (π_s) and Manufacturer's profit (π_m) are given as:

$$T_r = w.S(K) + p_o.(K - S(K)) - p_s.(\mu - S(K))$$

$$\pi_s = (w - p_o + p_s - c_p)S(K) + (p_o - c_k)K - p_s.\mu$$

$$\pi_m = (r - w + p_o - p_s)S(K) - p_o.K + p_s.\mu$$

Optimal capacity for supplier (K_s^*) and optimal capacity for manufacturer (K_m^*) will be:

$$\overline{F}(K_s^*) = \frac{c_k - p_o}{w - p_o + p_s - c_p}$$

$$\overline{F}(K_m^*) = \frac{p_o}{r - w + p_o - p_s}$$

Now, if we set the parameters meeting the following conditions:

$$\left.\begin{array}{r} \left(w - p_o + p_s - c_p\right) = \lambda.\left(r - c_p\right) \quad ......(10-a) \\ \left(c_k - p_o\right) = \lambda.c_k \quad ......(10-b) \end{array}\right\} \quad ......(10)$$

Then:

$$\left.\begin{array}{r} \pi_s = \lambda.\Pi - p_s.\mu \quad ......(11-a) \\ \pi_m = \left(1-\lambda\right).\Pi + p_s.\mu \quad ......(11-b) \end{array}\right\} \quad ......(11)$$

The condition (11) above would coordinate the supply chain when wholesale price and penalties are simultaneously decided. These results of special case are similar to what Cachon [3] has derived in capacity procurement game. However, when the wholesale price is exogenously determined, the model without penalty does not guarantee coordinate since wholesale price and buy-back price need to be determined simultaneously for coordination, and also, it would fail to provide flexibility in profit allocation between players, whereas, in this model, the two-way penalties would provide that flexibility.

5. Conclusion

In this paper we argue that the idea of infinite penalty does not make much sense, as its enforcement cannot be ensured, therefore, forced compliance is not a realistic assumption. On the other hand, voluntary compliance is a too conservative assumption. Therefore, we propose *price compliance* regime, where the parties comply on prices, rather than quantities. We model the supply chain in price *compliance regime* where investment in capacity building (or enhancement or updating the capacity) is a prerequisite for production and supplier needs to invest in capacity before the demand is realized. With the capacity and cost of building capacity as model parameters, we introduce two penalties, penalty for short supply and penalty for short orders, and one bonus, excess supply bonus in the model. We show that with capacity commitment in the form of target capacity, manufacturer can influence supplier's capacity decision. For the generic model, we show that supplier's profit function is unimodal, and this leads to characterization of her capacity decision.

We show the analytical results for coordination conditions for a special case when demand is uniformly distributed. For this special case, the illustration suggests there is a continuum of contracts which coordinates the supply chain with flexibility in profit allocation. For a given structure of penalties and bonus, we analyze how the target setting influences supplier's capacity decision, and hence supply chain coordination, and show that manufacturer can not influence supplier's capacity decision by setting higher target capacity unless he compensates her for any shortage in orders with respect to this target capacity. So, supply chain coordination can not be achieved without short order penalty in the contract.

We also present the special case where the capacity is visible and where both manufacturer and supplier jointly decide the capacity. We analytically show the conditions for coordination and illustrate that the presented structure of contract offers more flexibility in profit allocation between players.

The generic model presented here demonstrates an approach to supply chain coordination through contract with realistic and executable assumptions. Though, the analytical solution to generic model is not available, we present two special cases to demonstrate the potential of presented structure of contract in terms of achieving supply chain coordination and providing flexibility in allocating profit between the two players. We propose to extend this model for two asymmetric information scenarios, one, when supplier doesn't have the demand information, and second when manufacturer doesn't have information of supplier's cost structure.

Appendix

Appendix-1 : Proof for equation (4-a) and (4-b)

In the above expression (4-b), $K_s^* = T$ represents the boundary condition for the two scenarios. Scenario-1 has the lower boundary condition of $K = 0$ and supplier's profit at this boundary condition, $\pi_{s1}(K = 0)$ is negative. Since supplier's profit function π_{s1} is unimodal, as shown earlier, we can conclude that for any value of $T > 0$, $\pi_{s1}(K = 0) < \pi_{s1}(K = T)$. Hence, if K_{s1}^*, as per equation (2), obtained from unconstrained maximization of supplier's profit function in scenario-1 violates the prerequisite condition of the scenario, i.e., $K \leq T$, then the constrained optimal capacity for the scenario would be nothing but T.

Similarly, scenario-2 has the upper boundary condition of $K = \infty$, and supplier's profit at this boundary condition, $\pi_{s2}(K = \infty) = -\infty$. Since supplier's profit function π_{s2} also has been shown to be unimodal, can conclude that for any value of $T < \infty$, $\pi_{s2}(K = \infty) < \pi_{s2}(K = T)$. Hence, if K_{s2}^*, as per equation (3), obtained from unconstrained maximization of supplier's profit function in scenario-2 violates the prerequisite condition of the scenario, i.e., $K > T$, then the constrained optimal capacity for the scenario would be nothing but T.

In view of the above, let us examine all the possible scenarios:

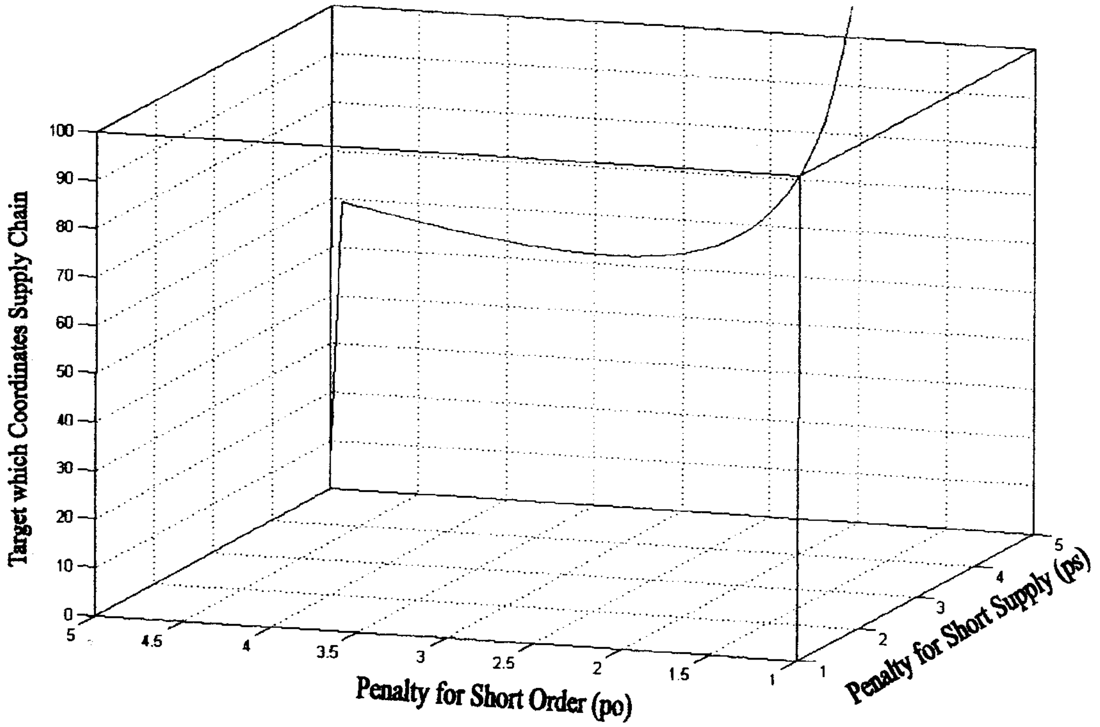
1. When $\pi^*_{s1} \ge \pi^*_{s2}$ & $K^*_{s1} \le T$: The supplier would build a capacity $K = K^*_{s1}$ since unconstrained maximization of supplier's profit function in scenario-1 does not violate the prerequisite condition of the scenario.
2. When $\pi^*_{s1} \ge \pi^*_{s2}$ & $K^*_{s1} > T$: The suppliers capacity decision would depend on her profit π_{s1} at boundary condition K = T since unconstrained maximization of supplier's profit function in scenario-1 has violated the prerequisite condition of the scenario.
    a. If $\pi^*_{s2} < \pi_{s1}(\mathrm{K} = \mathrm{T})$, she would build a capacity $K = T$
    b. If $\pi^*_{s2} \ge \pi_{s1}(\mathrm{K} = \mathrm{T})$, she would build a capacity $K = K^*_{s2}$
3. When $\pi^*_{s1} < \pi^*_{s2}$ & $K^*_{s2} > T$: The supplier would build a capacity $K = K^*_{s2}$ since unconstrained maximization of supplier's profit function in scenario-2 does not violate the prerequisite condition of the scenario.
4. When $\pi^*_{s1} < \pi^*_{s2}$ & $K^*_{s2} \le T$: The suppliers capacity decision would depend on her profit π_{s2} at boundary condition K = T since unconstrained maximization of supplier's profit function in scenario-2 has violated the prerequisite condition of the scenario.
    a. If $\pi^*_{s1} < \pi_{s2}(\mathrm{K} = \mathrm{T})$, she would build a capacity $K = T$
    b. If $\pi^*_{s1} \ge \pi_{s2}(\mathrm{K} = \mathrm{T})$, she would build a capacity $K = K^*_{s1}$
5. When $K^*_{s1} > T$ & $K^*_{s2} \le T$, she would build a capacity $K = T$

Hence, to summarize, we can say that supplier's response would be to install a capacity equal to any of the three values, K^*_{s1}, K^*_{s2} or T, which proves the equation (4-a), and conditions given in five scenarios, when summarized, lead to equation (4-b).

Appendix-2 : Illustration for Coordination conditions

The graph (refer figure-6) shows that for scenario-1 ($K \leq T$), there exists a continuum of contracts which coordinate the supply chain. The starting point of the graph, $p_s = p_o = 1$ represents the condition (8-a) where coordinating target $T = a = 100$. The coordinating target falls below a, as $(p_s + p_o)$ increase. All the points on the continuum represent one coordinating contract, each with different distribution of profits between the two parties.

Figure-6 : Target Setting for Coordination for different p_s and p_o

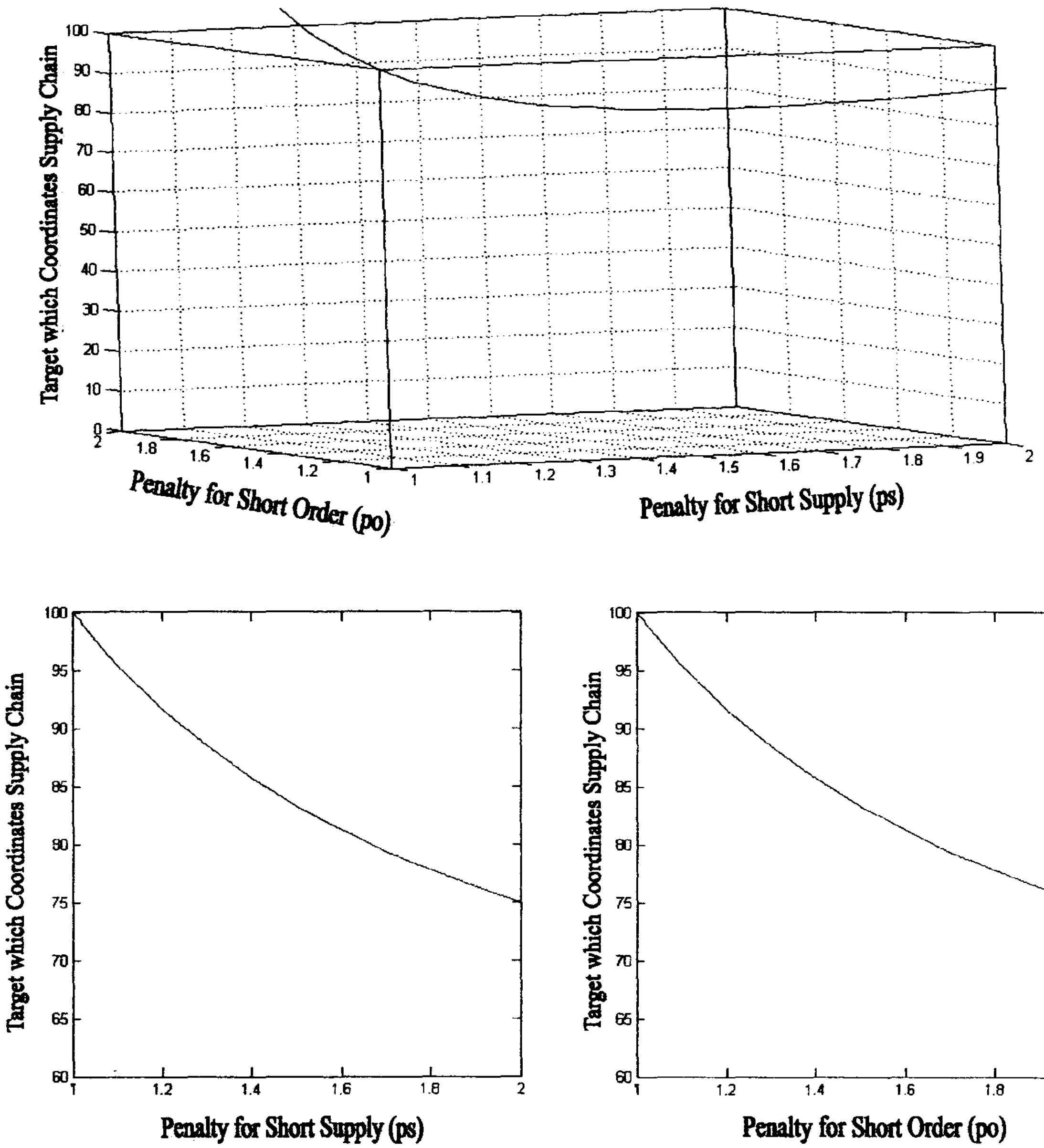


However, un-bounded penalties might not be negotiable in all situations. If we limit the penalties to the maximum possible loss in default condition, then:

$$\left.\begin{array}{l} p_s \le r - w \quad \ldots\ldots(12-a) \\ \\ p_o \le c_k \quad \ldots\ldots(12-b) \\ \\ p_b \le r - w \quad \ldots\ldots(12-c) \end{array}\right\} \quad \ldots\ldots(12)$$

In this illustration, when penalty parameters were subjected to the bounds given by condition (12), still there existed a feasible continuum of contract, which coordinates supply chain. The coordinating continuum with bounded penalties is given in the three-dimensional graph as well as projections on axes for ease of understanding (refer figure-7).

Figure-7 : Target Setting for Coordination for different p_s and p_o in bounded conditions



Appendix-3 : Contracts with bounded penalties, which improve Supply Chain Performance

The following table (refer table-1) lists a few contracts which coordinate the supply chain in scenario-1 ($K \leq T$). Performance of the same contracts in terms of profit allocation, efficiency and improvement over pure wholesale price contract is given in the next table (refer table-2).

Table-1

S. No.	Contract Parameters				Supplier's optimal Capacity Ks*	Supplier's Profit at Ks*	Manufacturer's Profit at Ks*	Supply Chain Profit at Ks*
	Ps	Po	Pb	T				
1	1.5	0.5	0	100	75	150	75	225
2	1.5	1	0	87.5	75	165.23	59.77	225
3	1	1	0	100	75	175	50	225
4	1.5	1.5	0	83.33	75	180.21	44.79	225
5	1	1.5	0	91.67	75	189.41	35.59	225
6	1.5	2	0	81.25	75	194.82	30.18	225
7	0.5	1.5	0	100	75	200	25	225
8	1	2	0	87.5	75	203.91	21.09	225
9	0.5	2	0	93.75	75	214.16	10.84	225
10	0	2	0	100	75	225	0	225

Table-2

S. No.	Contract Parameters				Supplier's share of profit	Manufacturer's share of profit	SC Efficience	Improvement in SC Performance
	Ps	Po	Pb	T				
1	1.5	0.5	0	100	66.67%	33.33%	100.00%	1.260%
2	1.5	1	0	87.5	73.44%	26.56%	100.00%	1.260%
3	1	1	0	100	77.78%	22.22%	100.00%	1.260%
4	1.5	1.5	0	83.33	80.09%	19.91%	100.00%	1.260%
5	1	1.5	0	91.67	84.18%	15.82%	100.00%	1.260%
6	1.5	2	0	81.25	86.59%	13.41%	100.00%	1.260%
7	0.5	1.5	0	100	88.89%	11.11%	100.00%	1.260%
8	1	2	0	87.5	90.63%	9.38%	100.00%	1.260%
9	0.5	2	0	93.75	95.18%	4.82%	100.00%	1.260%
10	0	2	0	100	100.00%	0.00%	100.00%	1.260%

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