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Optimal Deployment of Parallel Teams in New Product Development

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Abstract

Speed in product development can be defined as the thin line that determines success or failure in many fast growing industries today. Strategies like set based concurrent engineering and parallel team deployment have been used to accelerate new product introduction by successful companies like Toyota. In the current paper, these two strategies are combined to investigate the impact of generating sets of alternatives using parallel teams on the revenue generated from new product development. The trade-off between cost and speed of development due to deployment of parallel teams in product development using the Multi Armed Bandit framework are also explored. For sequential and overlapped development strategies, quantitative models to determine the optimum number of product development teams at different stages of development have been constructed using the Gittins Index strategy. These models have been tested empirically using hypothetical data and the results show that the higher flexibility induced quality due to larger set of design alternatives results in higher rewards, despite the higher cost of deploying multiple teams.

Key words: New Product Development; parallel teams; Multi Armed Bandit; Gittins Index; sequential development; overlapped development

Introduction

New product development can provide a sustainable competitive advantage in many industries, if carried out effectively, at the right time with the right cost and quality. In recent times, there has been significant emphasis on time to market in the new product development literature (Krishnan and Ulrich, 2001). Extensive studies have been carried out on how firms from various industries in Japan have overtaken established firms in the US in new product introduction by mastering the art of innovation (although there are counter examples in industries like software). A careful study of literature reveals that many Japanese companies allocate resources very differently from their American counterparts, and use multiple product development teams at certain stages of design. However, there is very little theoretical or empirical research carried out to test this phenomenon, except for a few analytical models and anecdotal references (Nelson, 1961, Arditti & Levy, 1980) by pioneers of new product development and Knowledge Management (Nonaka 1994 and Takeuchi 1986). However, the set-based concurrent engineering practices at Toyota substantiate the advantages and effectiveness of having design alternatives at the system, sub-system and part level (Ward *et al*, 1995). In this paper, the concept of parallel teams developing various design alternatives and prototypes during the different stages of development of a product is analyzed in the context of radical innovation. Quantitative models are developed to determine the optimum number of teams at each stage of product development for (i) Sequential development and (ii) Overlapped development strategies. The trade-off between product development time, cost and quality under parallel product development strategy are explored with the help of Multi Armed Bandit problems and a framework is developed to test these models with hypothetical data. The well-known Gittin's Index strategy is used to arrive at the optimum decision rule.

From the literature survey it is evident that the phenomenon of deploying parallel teams has been present in both the western as well as the eastern worlds, at least since the days of World War II. This strategy has been successfully used in cutting down time and improving quality in critical projects like aircraft engine development, missile development, production of self sustaining material in atomic bomb development etc. particularly in the war-time projects. Competition in today's market is almost war-like, and strategies like deployment of parallel teams may provide the necessary edge to overtake the competition and achieve a higher market share by significantly reducing the product development time without compromising on quality.

The construction of the optimal decision rule significantly contributes to the selection of the optimum number of teams at various stages of development under the general framework which assumes *time to development* as a random variable. For example, Nelson (1959) who has been credited with providing the first analytical framework for the use of parallel product development teams (the so called 'parallel path strategy'), states with the help of a number of real life examples that the cost of using several teams during the initial stages is much smaller when compared to the benefits that accrue from the information gathered. Based on this principle, his model assumes that n teams are selected to work on the project up to some review point and the team that looks best during the review, in terms of successfully completing the project at minimum cost, is retained, while the remaining n-1 teams are dropped. Although Arditti and Levy (1980) have extended Nelson's model to include the probability of success in determining the optimum number of teams, they also agree that their model needs further extensions. The proposed decision rule, which has been constructed on the basis of the Gittins strategy, helps in choosing the optimum number of teams at each stage of a project when the time required for development is a random variable and the number of attributes and development teams are innumerable. At the same time the model takes into account the dynamic information that becomes available as the development progresses, into the reward function calculations and hence the optimum decision rule, enabling the optimum use of resources under complex conditions.

2. Literature Survey

In this era of fast innovation and rapid obsolescence, the ability to introduce new products into the market at a rapid pace has become a necessity to survive global competition. The need for quick product development is equally touted both by the academic researchers and the industry practitioners. Empirical studies have revealed that, a 6 month delay in the launch of a new product can reduce overall earnings by up to 33% (Dumaine 1989). In industries like pharmaceuticals, it is imperative to introduce a new drug before the competitor does, as the patent protection otherwise would completely bar entry into the market and the huge R&D investments of the range 8-10 million dollars into that particular drug development would result in zero returns. Consequently, time based competition provides a sustainable competitive advantage that can increase profits and market share, while simultaneously containing costs and market risk (Page, 1993). There is both theoretical and empirical research that substantiates first-mover and second-mover advantages from fast innovation (Lieberman & Montgomery, 1988).

However, there are ample studies analyzing the disadvantages of first movers and the trade-off between quality and speed of innovation. Camel, (1995) and Smith and Reinertsen (1991) present evidence with regard to how overemphasis on speed of innovation and time to market may entail reducing performance specifications. Some researchers also pointed out the disadvantages of fast product development and pioneering new technologies (Lounamaa *et al*, 1987, Von Barun, 1990). There are hidden costs and downsides to speed in new product development, such as more mistakes, heavy usage of resources and disruptions in workflow etc. (Crawford (1992) and Von Barun (1990). The review paper by Krishnan and Ulrich (2001) analyses the literature on product development from a decision based perspective and lists a number of very relevant papers which focus on product development projects within a single firm. The current paper addresses the trade-off between speed of development, resource usage and quality levels by providing a framework to arrive at an optimum decision rule that while ensuring improved speed of development and a given quality level, also avoids unnecessary use of resources.

According to the Oslo Manual (OECD1992), the concept of technological innovation includes not just substantially new products which are designated as major innovations, but also significant technological changes to existing products and processes. Based on their magnitude, innovations are classified into three categories: Incremental, Standard and Radical. An improvement in the performance characteristics of already existing products is termed as incremental product innovation. On the other hand, radical innovation, which takes a substantial amount of deviation from the existing concepts, needs to fulfill the two necessary conditions, namely (i) an overturned core concept of the product and (ii) a major change in the linkage among the core components of the product (Henderson and Clark 1990). A standard innovation falls in between incremental and radical innovations, filling the large gap between the two, and is mostly disregarded by many in the literature on product development (Gordon 1992). The current paper focuses mainly on radical innovation, although the framework can be extended to include standard and incremental innovations too.

It is evident from a comprehensive review of the existing literature on new product development, that there is a strong need to find ways of ensuring the quality of product development while at the same time garnering the benefits of fast innovation. One country that has consistently achieved this combination of quality and speed of innovation, at least in the machinery industries is Japan. Bussey (1988) argues that the U.S manufacturers' lack of competitiveness with the Japanese in industries like photocopying machines and automobiles can be attributed to the prolonged product development cycles at the American companies. An empirical analysis carried out by Mansfield (1988) to test these anecdotal references suggests that Japanese companies in comparison to their American counterparts, typically spend much less on product development, but at the same time they innovate much faster, especially in industries like rubber, instruments, metals, electrical equipment and machinery (including computers). Two important reasons for this achievement of Japanese companies, unearthed from Mansfield's work are very striking and relevant to our present context. Firstly, the way Japanese allocate their resources to the innovation process; and secondly, their willingness to devote about twice as many resources to accomplish time reduction in innovation.

2.1 Japanese Strategies in New Product Development

Extensive research by Nonaka (1985 and 1994) in the fields of New Product Development and Organizational Knowledge Creation has led to various theories that analyse the sustained success of Japanese companies in this regard. According to him, Japanese companies manage the process of Organizational Knowledge Creation through *Creative Chaos, Redundancy* and *Requisite Variety*. *Redundancy* is especially emphasized during New Product Development, through conscious overlapping of company information, business activities and management responsibilities. He points out that, to Western managers, the term "redundancy" with its connotations of unnecessary duplication and waste may sound unappealing. Nevertheless, redundancy plays a key role in speeding up the process of concept creation, reducing managerial hierarchy and enhancing mutual trust between the members of the product development team. Redundancy is built into the process of product development by dividing the product development team into competing groups that develop different approaches to the same project and then debate the advantages and disadvantages of each other's ideas before agreeing upon the best approach. Internal rivalry encourages the team to look at a project from varied perspectives. Having parallel teams at the concept development stage ensures high design quality at minimum time.

A deeper search of the New Product Development literature reveals that the practice of employing parallel development teams is not an exclusive Japanese strategy, but one that was very much present in the western world as well. For example, in the early development stages of the atomic bomb, five independent teams at various universities pursued the problem of producing large quantities of material capable of producing a self-sustaining reaction. A similar strategy was employed in the development of aircraft engines during the Second World War, as it was found that the best and the fastest way to produce a good engine was by using parallel teams to develop several prototypes and then choosing the best prototype and work on (Robert and Heron, 1950). There are plenty of other instances wherein this strategy of competing parallel research teams has been put into use effectively, like when the US developed the Thor and the Jupiter missiles and the Soviet Union developed the MIG fighters (Nelson 1959). In more recent times, in industries like software, outsourcing of a few segments of product development to low cost destinations such as India has become quite prevalent. Due to the low cost of product development skills, it is quite common in this field to employ parallel teams to write multiple programs for the same segment at offshore locations (usually overnight), while the more expensive onshore team shortlists the most efficient program (with the least number of bugs) for each segment next day morning in order to reduce the time consuming product development time. Exploration of innumerable leads for a potential molecule in the initial stages of drug development is quite common in the pharmaceutical industry as well and the Multi Armed Bandit framework is in fact quite extensively used to determine optimal strategies at various stages of new drug development.

However, having redundant resources like product development teams can be very expensive as high-quality product development employees are a scarce resource, and one has to always outweigh the benefits of quality versus cost. When high profile products of great market potential are being developed, it makes sense to expend on quality and speed, which might eventually pay off, once the product is introduced into the market at the right time. Having multiple designs developed by different product development teams can be quite useful at the critical junctures of product development, especially when practices like concurrent engineering are followed.

Toyota's development process, the so . called "Set-Based Concurrent Engineering" (SBCE) overcomes some of the problems faced in the classical concurrent engineering approach. In SBCE, designers explicitly communicate and think about sets of design alternatives at both the conceptual and the parametric levels. They gradually narrow these sets by eliminating inferior alternatives until they come to a final solution. Thus, due to the rigorous efforts put in the initial stages of development, the rework in the upstream stages is substantially reduced once the design progresses to the down stream stages; at the same time flexibility and quality through multiple designs at both the system and subsystem levels are ensured. *By communicating a whole set of possibilities simultaneously and avoiding changes that move outside the set, Toyota is able to reduce the frequency of communication, eliminate the need for dedication and collocation of product development teams and reduce supplier communications* (Ward *et al*, 1995).

Toyota's development process is radically different from the practices in the U.S, Europe or even in other plants of Japan's auto industry. Most of the western designers follow *Shigley's Mechanical Engineering Design*, a hill climbing process that iterates through a sequence of steps: (i) Understand the problem, (ii) Synthesize a solution, (iii) Analyze and evaluate the solution, and (iv) Based on the evaluation, try for a new solution. Here, the emphasis is on "speeding up the iterative loop" by "reducing the number of iterations" and "doing it right the first time" or "freezing the specifications early". However, in the case of Toyota, they follow a completely different set of rules. These are (Ward *et al*, 1995):

(i) Explore a larger number of concepts in 1/4 or 1/5 scale clay models and expend more resources

(ii) Delay fixing key dimensions that determine body shape

(iii) Delay releasing final specifications to suppliers until relatively late in the design process

(iv) Develop prototypes of an extraordinary number of different designs for subsystems

In the current paper, models for deployment of parallel product development teams are developed for (i) traditional sequential development and (ii) overlapped development approaches using the multi armed bandit framework. Note that 'overlapped development' approach has many similarities with Toyota's SBCE and is more generic in nature as it encompasses both sequential and concurrent development phases.

3. Research Methodology - Multi Armed Bandit (MAB) Framework

The Multi Armed Bandit (MAB) framework was first introduced in clinical trials by Thompson (1933), and was subsequently used extensively used by others in clinical drug trials, petroleum exploration, consumer product or service settings and pricing strategies(e.g., Gittins (1989), Meyer and Shi (1995) and Erdem and Keane (1996), Bergemann and Valimaki (1996)). Rothschild (1974) was amongst the first economists to introduce the MAB framework into economic theory by bringing in imperfect knowledge of market conditions into the decision making process of firms and managers while choosing price and output decisions. The framework of bandit problems has also proved popular for the analysis of decision making in labor markets, notably in the theory of job-search and matching (Mortensen 1985). Gittins has contributed significantly to the theory of MAB by developing the 'Gittins Index Strategy' which is extensively used in many practical applications in various fields (Gittins, 1979).

The name "bandit" comes from modeling these problems as a multi armed bandit, which is a slot machine with multiple arms, each yielding an unknown and possibly different distribution of payoffs. Gittins and Jones (1974) proved that the k-armed bandit problems can be solved by solving k-one armed bandit problems. This theorem asserts that in any independent-armed bandit problem with geometric discounting over an infinite horizon, it is possible to associate with each arm an index, known as the Gittins index, with the property that a strategy in the bandit problem is an optimal strategy if, and only if, it (almost) always involves playing an arm with the highest value of the Gittins index at that point. (Such a strategy is called the Gittins index strategy.) The feature that gives this result especial potency is that the Gittins index on an arm depends solely on the characteristics of that arm and on the rate of discounting, but not on any other feature of the problem under study (Sundaram, 2003). Although the Gittins theorem seems more restrictive due to the independent arms requirements, there are many other results on bandit problems, which do not require such restrictive assumptions. For example, Karoui and Karatzas (1997) have proved that an optimal strategy can be determined for multi-armed bandit problems with arms that are not necessarily independent or Markovian, using notions and results from time-changes, optimal stopping and the multi-parameter Martingale theory. Ansell *et al* (2003) have formulated a discounted costs version of the server control problem as a *restless bandit* problem with dependent arms. They used Whittle's (1988) index-based heuristic approach to develop the required indices and proved with a numerical study that their index policy in fact shows a strong performance.

It is quite difficult to model various factors like quality, speed, flexibility, cost of development etc. simultaneously and determine an optimum decision rule that ultimately maximizes the total reward in a dynamic environment, using traditional optimization techniques. The bandit framework not only allows for the modeling of all these factors but is also comprehensive enough to handle various widely used new product development practices like sequential, overlapped and parallel development.

Due to the above mentioned reasons, the Multi Armed Bandit framework has been adopted in the current paper to solve the selection of an optimum number of teams.

In the proposed research on parallel product development teams, it has been assumed that each arm in the bandit framework represents a product development scenario with 'i' teams, where i = 1,2...n. We divide the entire development work into m development stages based on different attributes that have sequential dependence. In other words, the development of attribute 2 cannot begin before the development of attribute 1 starts, and also, it cannot be completed before the development of attribute 1 is completed and so on[1]. Thus, we have a decision problem, with *n* possible actions at each stage where action *k* in stage *j* results in employing *k* number of teams for the development of the j^{th} attribute[2]. Each arm is associated with a reward, which is the expected revenue from the development of that attribute when the product is introduced into the market after subtracting the cost of development. The revenue from an attribute depends on the time taken for the development and quality of the attribute, as well as the quality of the overall product after development, apart from the market value of the attribute. We define the quality of the overall product after development as the "flexibility of the interface each developed attribute has contributed to the product as a whole". This particular factor is very critical in the process of New Product Development, as most often, it is the trade-off between the different attributes or the cumulative value of various attributes of a product, that determines the market value of the product, rather than a single attribute alone. Thus, the flexibility in specifications of an attribute until late into the development, which allows for the development of other attributes without compromising on their specifications, becomes very important[3]. Flexibility in our context is assumed to be the number of design alternatives or prototypes that are independently developed by parallel teams. The reward R_j corresponding to each attribute then is a function of cost of development, time to development and the number of teams 'i' dedicated to the development of attribute j. We also assume a discounting factor β, which is used in calculating the net present value of the total reward at the end of the development.

[1] Note that sequential dependence does not mean the development approach has to be necessarily sequential in nature. In fact, we use both sequential and overlapped development approaches in the current paper.

[2] Here we assume that all *k* teams are working in parallel on the same attribute (*j*), and they share their design alternatives whenever there is a need to freeze the specifications, like at the end of each stage.

[3] It is to avoid the *quality loss* discussed by Krishnan *et al* (1997), that occurs due to the loss of flexibility to make future changes in the upstream activities, once the information is passed on to the downstream stages.

3.1 Multi armed Bandit Formulation

Let us consider the development of m attributes, assuming that the development is in the order j = 1,...,m, where each attribute is set in the order of its sequential dependence. The period j of MAB for example is devoted to the development of the j^{th} attribute. Our objective is *to find an optimal decision rule, that is, the optimal number of parallel teams to be deployed in each period in order to maximize the net present value of the total cumulative reward from all the periods of the product development.*

We formulate the Parallel Product Development Teams as a Multi armed Bandit problem with the following Notation:

$i \in \{1,2,...n\}$ denotes a finite set of "arms" of the bandit, where i denotes the number of product development teams employed for the development of attribute j using arm 'i'.

c_j^i denotes the random variable representing cost of developing attribute j using 'i' product development teams.

$T_j, j = 1,2,...,m$ is the maximum stipulated time allowed for development of attribute j.

τ is a random variable representing the '*time to development*', whose prior probability distribution at time t=0 is assumed to be known for all arms, i=1,...,n and the posterior distributions are determined dynamically as and when new information becomes available about a given arm.

$t_j^i, j = 1,2,...,m$ is the time spent on improving the attribute j by a given magnitude using 'i' product development teams and is the expected value of the random variable τ.

$r_j^i(\tau), j = 1,2,...,m$ is the random variable representing the revenue obtained after the development of attribute j using arm 'i' and is a function of *time to development*, τ.

$p_j^i \in [0,1]$ is the probability of successful development of attribute j using 'i' development teams

$\beta \in (0,1)$ is the discount factor, assumed to be a constant[4].

Assumptions:

(i) The expected development time t_j^i for each attribute j depends on the arm 'i' (i.e., the number of product development teams) and influences the market value of the product at the end of development.

(ii) The teams within an arm cooperate with each other, meaning, they share their design alternatives whenever there is a need to find a compatible design for the upstream stages once the development in the downstream stages is completed, in order to avoid rework and continue with the consented new specifications in the next stages of development.

(iii) The probability of success p_j^i depends on the number of parallel teams used for development.

(iv) While calculating the revenue r_j^i, it is assumed that all costs to be incurred in the future production of the product are accounted for, except for the costs of development c_j^i.

3.2. The Reward Function

The expected reward, $E(R_j^i)$, for attribute j using arm i (that is, using i teams) can be computed by subtracting mean expected cost from the mean revenue and incorporating probability of successful development. Assuming q_1 to be the probability of failure corresponding to attribute 1, the expected reward for a single product development team from the development of attribute 1 is:

[4] Please note that the discount factor can be a random variable, and can be modeled using the MAB framework, however for the sake of simplicity, we assume it to be a constant in the current paper

$$E(R_1^1) = (1 - q_1)E(r_1^1) - E(c_1^1)$$

where $E(r_1^1)$ and $E(c_1^1)$ are the expected revenue and expected cost respectively corresponding to the development of attribute '1' using a single product development team.

Since the probability of successful development using two teams is the same as one minus the probability that both teams have failed, (i.e.,$(1 - q_1^2)$), the reward using two-development teams (corresponding to attribute '1') is given by

$$E(R_1^2) = (1 - q_1^2)E(r_1^2) - E(c_1^2)$$

where $E(r_1^2)$ and $E(c_1^2)$ are the expected revenue and expected cost associated with the development of attribute '1' using two product development teams. Please note that the superscript in term q_1^2 on R.H.S of equation (13) corresponds to 'square' unlike in r_1^2 and c_1^2, where the superscript represents the number of teams used for development. In general the expected reward for successful completion of attribute '1' using 'i' product development teams is given by

$$E(R_1^i) = (1 - q_1^i)E(r_1^i) - E(c_1^i) \tag{1}$$

where again superscript in the q_1^i on R.H.S of equation (14) corresponds to 'i^{th} power'.

Now, since we are assuming that the development of each successive attribute depends on the successful development of its previous attribute, the reward for attribute 'j' will depend on the successful development of previous rewards, 1,2,...,(j-1) and the expected value of reward for attribute 'j' may be written, using equation (1) as

$$E(R_j^i) = (1 - q_1^i)(1 - q_2^i)\ldots(1 - q_j^i)E(r_j^i) - E(c_j^i) \tag{2}$$

We formulate the new product development problem in forthcoming Section 3.5 and discuss the application of Gittins Index Strategy to the cases of sequential and overlapped development in

Section 3.6. However, before that we prove the following Lemma 1 and Theorems 1 & 2 which will later be used to show that the necessary conditions for application of Gittins Index Strategy to both these cases are satisfied.

Lemma 1

Assuming the *time to development* of a given attribute follows exponential distribution, the mean time to develop the attribute does not exceed the maximum allowed stipulated time, say T, if and only if the rate of development is greater than or equal to 1/T.

Proof

Let T_j be the maximum allowed stipulated time to development for arm j, and let the random variable τ denote the time arm *i* takes to develop attribute *j*. Since *time to development*, in the new product development literature is in general assumed to follow exponential distribution, we assume that the random variable τ follows exponential distribution, with parameter $\lambda_j^i = i * \lambda_j$, where λ_j is the parameter corresponding to jth attribute with single team. Let us first assume that mean time to develop the attribute *i* for any arm *j* does not exceed T_j.

$$\Rightarrow E(\tau) \le T_j \Rightarrow E(\tau) = \frac{1}{\lambda_j^i} \le T_j \Rightarrow \lambda_j^i \ge \frac{1}{T_j} \tag{3}$$

Thus the rate of development λ_j^i is greater than or equal to 1/ T_j. On the other hand, assuming the rate of development λ_j^i is greater than or equal to 1/ T_j, we have

$$\lambda_j^i \ge \frac{1}{T_j} \Rightarrow \frac{1}{\lambda_j^i} \le T_j \Rightarrow E(\tau) \le T_j \tag{4}$$

3.3. Sequential development

For the sake of simplicity we first consider the case of sequential development, where product development starts with a single attribute *j* (=1, say) in period t = 1 (or stage 1). At the beginning of period t = 2 (or stage 2), the development of next attribute in the sequence begins and so on until the development of all m attributes is completed.

Theorem 1

Assuming sequential development and that the estimated revenue Z_j and mean development cost $E(c_j)$ associated with a single team corresponding to attribute j are bounded, the sequence of rewards $\{R_1^i, R_2^i, ..., R_m^i\}$, $i = 1,2...n$ corresponding to attributes j = 1,...m are uniformly bounded if the rate of development during the stipulated time period T is greater than or equal to 1/T.

Proof:

Let Z_j be the estimated revenue from attribute 'j' if the development of the attribute is complete at the beginning of period j (Z_j is essentially the current market value of attribute j, if the product is immediately available for sale). However - without loss of generality - we can assume that the revenue will reduce as the time passes and ultimately becomes zero beyond a time point, say T_j. Then, we can define the revenue r_j^i, corresponding to jth attribute with i teams as a function of random variable τ as follows:

$$r_j^i(\tau) = Z_j * \left(\frac{T_j - \tau}{T_j} \right)$$

where $0 \le \tau \le T_j$ and the probability density function of the random variable τ, corresponding to arm i and attribute j is given by $f_j^i(\tau) = \lambda_j^i e^{-\lambda_j^i \tau}$.

Therefore the mean of the random variable τ corresponding to arm i and attribute j is given by $E(\tau) = \frac{1}{\lambda_j^i} = \frac{1}{i * \lambda_j}$, which shows that, as number of teams i increases, the average time needed to develop a given attribute reduces[5]. The expected revenue E(r) can be derived as follows:

$$E(r_j^i) = \int_{-\infty}^{\infty} r_j^i(\tau) f_j^i(\tau) d\tau$$

$$= \int_0^{T_j} Z_j * (\frac{T_j - \tau}{T_j}) * \lambda_j^i e^{-\lambda_j^i \tau} d\tau$$

$$= Z_j \left[\int_0^{T_j} \lambda_j^i e^{-\lambda_j^i \tau} d\tau - \frac{1}{T_j} \int_0^{T_j} \tau * \lambda_j^i e^{-\lambda_j^i \tau} d\tau \right]$$

$$= Z_j [1 - e^{-\lambda_j^i \tau}] - \frac{Z_j}{T_j} \left[-T_j * e^{-T_j \lambda_j^i} - \frac{1}{\lambda_j^i} [e^{-\lambda_j^i T_j} - 1] \right]$$

$$= Z_j \left[1 + \frac{1}{T_j \lambda_j^i} (e^{-T_j \lambda_j^i} - 1) \right] \quad (5)$$

For the sake of simplicity, we assume that the expected cost of a product development team - can be estimated based on the data from earlier projects - is equal to $E(c_j)$ for a single team and for i teams it is equal to $E(c_j^i)$, where

$$E(c_j^i) = i * E(c_j) \quad (6)$$

Substituting (5) and (6) in (2), we obtain the following expression for reward function:

[5] Here the implicit assumption is that, when a single team is used, a single prototype is produced, with narrow specifications. However, deployment of parallel teams assumes that there is scope for broader specifications and hence quicker prototype development by the parallel teams.

$$E(R_j^i) = (1-q_1^i)(1-q_2^i)\ldots(1-q_j^i)Z_j\left[1+\frac{1}{T_j\lambda_j^i}(e^{-T_j\lambda_j^i}-1)\right]-i*E(c_j) \quad (7)$$

We also have $0 < e^{-T_j\lambda_j^i} < 1 \Rightarrow -1 < (e^{-T_j\lambda_j^i}-1) < 0$ (8)

If the rate of development is greater than or equal to 1/T, for any attribute 'i' and arm 'j', from the above Lemma and inequality (4) we have,

$$E(\tau) = \frac{1}{\lambda_j^i} \le T_j \quad \Rightarrow \quad T_j\lambda_j^i \ge 1 \quad (9)$$

Inequalities (8) and (9) together imply

$$-1 < \frac{(e^{-T_j\lambda_j^i}-1)}{T_j\lambda_j^i} < 0$$
$$\Rightarrow 0 < \left\{1+\frac{(e^{-T_j\lambda_j^i}-1)}{T_j\lambda_j^i}\right\} < 1 \quad (10)$$

Since probabilities lie between 0 and 1,

$$0 < (1-q_1^i)(1-q_2^i)\ldots(1-q_j^i) < 1 \quad (11)$$

Since the revenue Z_j and cost $E(c_j^i)$ are assumed to be bounded, using inequalities (10) and (11) in equation (7), we can prove that the first moments exist for the sequences $\{R_1^i, R_2^i, \ldots\}$, $i = 1,2\ldots n$ and are uniformly bounded, i.e.,

$$\sup_{i\ge1,j\ge1} E\left|R_j^i\right| < \sup_{i\ge1,j\ge1}\left|E(R_j^i)\right|$$
$$= \sup_{i\ge1,j\ge1}\left|(1-q_1^i)(1-q_2^i)\ldots(1-q_j^i)Z_j\left[1+\frac{1}{T_j\lambda_j^i}(e^{-T_j\lambda_j^i}-1)\right]-i*E(c_j)\right|$$
$$< \infty$$

Hence the proof.

3.4. Overlapped Development

We next consider overlapped development of different attributes, j = 1...m, where each attribute is set in the order of sequential dependence. There are two parts to the overlapped development: (i) independent development and (ii) concurrent development of attributes at various stages of development depending upon the availability of required information from upstream stages to downstream stages. Please note that, since we are considering sets of solutions for each attribute, the concurrent part of overlapped development in the current context is similar to that of Toyota's SBCE, only here, a separate team would be deployed to develop each design alternative of a specific attribute. Similar to the overlapped development approach considered in the current paper, concurrent development in both traditional concurrent engineering (CE) as well as Toyota's SBCE, does not necessarily mean that the development of all m attributes will begin at the same time and end at the same time. Development of two consecutive attributes may either begin at the same time or the second attribute may begin a little while after the development of the first attribute begins, depending on the amount of information required and when it becomes available. Thus, the development of different attributes is more overlapping in nature than parallel. For the sake of simplicity and generalizability, we assume that the development of some attributes may begin at the same time, while others begin after some initial information becomes available. We also assume that, development of all attributes continues concurrently say up to time T_c, after which, development of attributes j+1,...,m cannot proceed unless development of attribute j (where j = 1,...,m-1) is completed, due to the sequential dependence. Let $T_j \geq 0$ be the excess time required[6] to complete the jth attribute independently after the initial concurrent development time T_c plus the independent development of first j-1 attributes. We consider these independent development periods as the periods in the MAB framework and treat them in the

[6] Note that $T_j = 0$ takes care of the possibility that jth attribute may not need any excess time for independent development.

same manner as that of sequential development described above, except for a few caveats discussed below.

Let Z_j be the estimated revenue from attribute 'j', if the development of attribute 'j' had been complete at the start of development of all attributes, i.e., at time t = 0. However, we can assume that the expected revenue reduces as the time passes and ultimately becomes zero beyond a point of time, say T. Thus the entire New Product Development, including the concurrent and independent development of attributes, has to lie between [0,T] and management will strive to reduce the total development time to a fraction of T using a variety of strategies, in order to maximize the revenue from the new product.

We define the revenue r_j^i, corresponding to arm 'i' and attribute j, as a function of random variable τ as follows:

$$r_j^i(\tau) = Z_j * \left(\frac{T-\tau}{T}\right)$$

where $0 \le \tau \le T_j$ and the probability density function of the random variable τ, corresponding to arm i and attribute j is

$$\begin{aligned} f_j^i(\tau) &= \lambda^i e^{-\lambda^i \tau} && \text{for } 0 \le \tau \le T_c \\ &= 0 && \text{for } T_c < \tau \le T_{j-1} \\ &= \lambda_j^i e^{-\lambda_j^i \tau} && \text{for } T_{j-1} < \tau \le T_j \end{aligned}$$

Here the parameter $\lambda^i = i * \lambda$, where λ is the rate of development during the concurrent development phase and $\lambda_j^i = i * \lambda_j$, where λ_j is the rate of development during the independent development period of each attribute j. Thus, following the Lemma in section 3.2, we have

$$\begin{aligned} E(\tau) &\le T_c && \text{for } 0 \le \tau \le T_c \\ &\le T_j && \text{for } T_{j-1} < \tau \le T_j \end{aligned} \qquad (12)$$

We can now prove the following theorem for overlapped development using the above notation and equation (12).

Theorem 2

Assuming overlapped development and the estimated revenue Z_j and mean development cost $E(c_j)$ associated with a single team corresponding to attribute j are bounded, the sequence of rewards $\{R_1^i, R_2^i, ..., R_m^i\}$, $i = 1,2...n$ corresponding to attributes j = 1,...m are uniformly bounded, if the rate of development during the time period T is greater than or equal to 1/T.

Proof: Using the above description of overlapped development and corresponding notation, the expected revenue from attribute j, using arm i can be calculated as follows:

$$E(r_j^i) = \int_{-\infty}^{\infty} r_j^i(\tau) f_j^i(\tau) d\tau$$

$$= \int_0^{T_c} Z_j * (\frac{T-\tau}{T}) * \lambda^i e^{-\lambda^i \tau} d\tau + \int_{T_c}^{T_{j-1}} Z_j * (\frac{T-\tau}{T}) * f(\tau) d\tau + \int_{T_{j-1}}^{T_j} Z_j * (\frac{T-\tau}{T}) * \lambda_j^i e^{-\lambda_j^i \tau} d\tau$$

$$= Z_j \left[\int_0^{T_j} \lambda^i e^{-\lambda^i \tau} d\tau - \frac{1}{T} \int_0^{T_j} \tau * \lambda^i e^{-\lambda^i \tau} d\tau \right] + 0 + Z_j \left[\int_{T_{j-1}}^{T_j} \lambda_j^i e^{-\lambda_j^i \tau} d\tau - \frac{1}{T} \int_{T_{j-1}}^{T_j} \tau * \lambda_j^i e^{-\lambda_j^i \tau} d\tau \right]$$

$$= Z_j \left\{ \underbrace{\left[1 - (1 - \frac{T_c}{T}) e^{-\lambda^i T} + \frac{1}{\lambda T} (e^{-\lambda^i T_c} - 1) \right]}_{(i)} + \underbrace{\left[\left(1 - \frac{T_{j-1}}{T} - \frac{1}{\lambda_j^i T} \right) e^{-\lambda_j^i T_{j-1}} - \left(1 - \frac{T_j}{T} - \frac{1}{\lambda_j^i T} \right) e^{-\lambda_j^i T_j} \right]}_{(ii)} \right\}$$

Substituting $E(r_j^i)$ in equation (2) we obtain expression for reward function:

$$E(R_j^i) = (1-q_1^i)(1-q_2^i)\ldots(1-q_j^i)Z_j \left\{ \begin{aligned} &\underbrace{\left[1-(1-\frac{T_c}{T})e^{-\lambda^i T} + \frac{1}{\lambda^i T}(e^{-\lambda^i T_c}-1) \right]}_{(i)} \\ &+ \underbrace{\left[\left(1-\frac{T_{j-1}}{T}-\frac{1}{\lambda_j^i T}\right)e^{-\lambda_j^i T_{j-1}} - \left(1-\frac{T_j}{T}-\frac{1}{\lambda_j^i T}\right)e^{-\lambda_j^i T_j} \right]}_{(ii)} \end{aligned} \right\} - i * E(c_j)$$

(13)

Since the remaining terms in equation (13) are already shown to be bounded in theorem 1 of sequential development, we only need to show that terms (i) and (ii) are bounded to prove theorem 2. First we begin with term (i):

We already know that

$$\begin{aligned} &0 \le T_c \le T \Rightarrow 0 \le \frac{T_c}{T} \le 1 \Rightarrow 0 \le (1-\frac{T_c}{T}) \le 1 \\ &\qquad \Rightarrow 0 \le (1-\frac{T_c}{T})e^{-\lambda^i T_c} \le 1 \qquad \text{since } e^{-\lambda^i T_c} \le 1 \end{aligned} \tag{14}$$

We can also show that

$$\begin{aligned} &e^{-\lambda^i T_c} \le 1 \Rightarrow -1 \le (e^{-\lambda^i T_c} - 1) \le 0 \\ &\qquad \Rightarrow -1 \le \frac{1}{\lambda^i T} * (e^{-\lambda^i T_c} - 1) \le 0 \qquad \text{since } \frac{1}{\lambda^i T} \le 1 \\ &\qquad \Rightarrow 0 \le 1 + \frac{1}{\lambda^i T} * (e^{-\lambda^i T_c} - 1) \le 1 \end{aligned} \tag{15}$$

Thus using inequalities (14) and (15) in (i), we can show that

$$|(i)| = \left\| \left[1-(1-\frac{T_c}{T})e^{-\lambda^i T} + \frac{1}{\lambda^i T}(e^{-\lambda^i T_c}-1) \right] \right\| \le 2 \tag{16}$$

We now consider (ii):

Since T is the maximum time available beyond which the revenue from new product becomes zero, we can assume that $T_c \le T_1 \le T_2 \le \ldots T_m < T$. Thus, we have

$$0 \le T_{j-1} \le T_j < T \Rightarrow 0 \le \frac{T_{j-1}}{T} \le \frac{T_j}{T} < 1$$

$$\Rightarrow 1 \ge (1 - \frac{T_{j-1}}{T}) \ge (1 - \frac{T_j}{T}) > 0$$

$$\Rightarrow 1 - \frac{1}{\lambda_j^i T} \ge (1 - \frac{T_{j-1}}{T} - \frac{1}{\lambda_j^i T}) \ge (1 - \frac{T_j}{T} - \frac{1}{\lambda_j^i T}) > -\frac{1}{\lambda_j^i T}$$

However, we also know that $0 < \frac{1}{\lambda_j^i T} \le 1 \Rightarrow 0 > -\frac{1}{\lambda_j^i T} \ge -1$

$$\Rightarrow 1 > (1 - \frac{T_{j-1}}{T} - \frac{1}{\lambda_j^i T})e^{-\lambda_j^i T_{j-1}} \ge (1 - \frac{T_j}{T} - \frac{1}{\lambda_j^i T})e^{-\lambda_j^i T_j} > -1$$

$$(\text{since } \lambda_j^i T_{j-1} \le \lambda_j^i T_j \Rightarrow 1 \ge e^{-\lambda_j^i T_{j-1}} \ge e^{-\lambda_j^i T_j})$$

$$\Rightarrow |(ii)| = \left| (1 - \frac{T_{j-1}}{T} - \frac{1}{\lambda_j^i T})e^{-\lambda_j^i T_{j-1}} - (1 - \frac{T_j}{T} - \frac{1}{\lambda_j^i T})e^{-\lambda_j^i T_j} \right| < 2 \qquad (17)$$

Thus, using the assumptions that Z_j and $E(c_j)$ are bounded and substituting (11), (16) and (17) in (13), we can prove that the first moments exist for the sequences $\{R_1^i, R_2^i, ...\}$, $i = 1,2..n$ and are uniformly bounded, i.e.,

$$\sup_{i \ge 1, j \ge 1} E\left|R_j^i\right| < \sup_{i \ge 1, j \ge 1} \left|E(R_j^i)\right| \le \left|4 * (1-q_1^i)(1-q_2^i)\dots(1-q_j^i)Z_j\right| + \left|i * E(c_j)\right|$$
$$< \infty$$

Hence the proof.

The revenue and cost functions r_j^i and c_j^i, in case of both sequential development and overlapped development are functions of *time to development* τ and the number of teams used for development and depend on the internal dynamics between the collaborating teams within an arm and hence are assumed to be independent for $i = 1,\dots,n$. Hence we can assume without loss of generality that $\{ R_j^i, j = 1,2,\dots,m\}$ is a sequence of independent random variables for $i = 1,\dots n$.

3.5 Problem Formulation

Our objective is to find an optimum decision rule $A = (a_1, a_2, ...)$, where a_k denotes the arm chosen at k^{th} stage that maximizes the total discounted reward:

$$V = \sum_{j=1}^{m} \beta^{j-1} E(R_j^{a_j}) = \max_{i=1,\cdots,n} \sum_{j=1}^{m} \beta^{j-1} E(R_j^i) \tag{18}$$

In order to find this optimum decision rule, the Gittins Index strategy is used (Varaiya, Walrand and Buyukkoc 1985) under the more general probabilistic structure described below: Given (i) the arms are independent, i.e., the sets of revenues from each arm, $\{R_1^1, R_2^1, ...\}, ..., \{R_1^n, R_2^n, ...\}$ are independent sets of random variables, (ii) $Sup_j E(R_j^i) < \infty$ for all $i = 1, ..., n$ and (iii) $\sum_1^{\infty} \beta_j < \infty$, where the discount factor $\beta \in [0,1)$; the Gittins index (Dynamic Allocation Index, Gittins 1979) for i^{th} arm may be calculated as follows:

$$G^i = \sup_{N \geq 1} \sum_{j=1}^{N} \beta^{j-1} E(R_j^i) / \sum_{j=1}^{N} \beta^{j-1} \qquad \text{for } i = 1, ..., n \tag{19}$$

According to the theorem of Gittins and Jones (1974), in the above setting (i.e., for the n-armed bandit with geometric discount and independent arms), it is optimal at each stage to select the arm with the highest Gittins index. They also prove that for any given bounded sequence of rewards $\{R_1^i, R_2^i, ...\}$, there exists a *stopping rule* N* such that the Gittins index corresponding to this sequence of rewards is attained at N* and is given by

$$G^i = \sum_{j=1}^{N^*} \beta^{j-1} E(R_j^i) / \sum_{j=1}^{N^*} \beta^{j-1} \tag{20}$$

Varaiya *et al* (1983) describe with the help of a theorem and corresponding lemmas which are a consequence of Gittins *stopping rule*, that once an arm turns out to be optimum (say in stage 1) with N* as the stopping rule, it turns out to be optimum at least up to the next N* stages. Thus, for an n – armed Bandit problem we construct the following *optimum decision rule* A with the help of Gittins theorem 3.3.1 and the stopping rule given by equation (20).

3.6 Optimum Decision Rule

Let N* denote the stopping rule that achieves the supremum for the Gittins index corresponding to arm k in stage 1 and let T^* denote the stopping rule corresponding to arm l for the highest Gittins index in stage N*+1 and so on, then decision rule A may be defined as follows:

(a) Use arm k in stages 1, 2, . . . ,N^*, and then if $N^* < \infty$,

(b) Use arm l in stages $N^*+1, \ldots, T^*$, and then if $T^*<\infty$,

(c) Continue according to A from stage T^* onwards.

Since our formulation of parallel product development teams (as arms of MAB) satisfies the conditions (i) – (iii) of Gittins Theorem, by choosing the highest Gittins index at each stage (i.e., for the development of each attribute), we arrive at a decision rule $A = (a_1, a_2, \ldots)$, which gives us the optimum number of teams to be chosen at each stage j that maximizes the total discounted reward given by (18).

4. Application of Gittins Index Strategy to choose optimum number of arms

In this section, the sequential and overlapped development models developed using Gittins Index strategy are applied on a sample of hypothetical data to determine the optimum decision rule. The data is given in Table 1 below, with the assumption that there are 5 attributes (j=5) and 5 arms (i = 1, ...,5) with arm 1 corresponding to single product development team, arm 2 corresponding to two product development teams and so on. The expected cost of development is given for a single team and is assumed to be a multiple of number of teams for different arms.

Attribute (j)	Maximum Revenue (Z)	Probability of success (p)	Time to development (T)	Discount factor (β)	Distribution Parameter (λ)	Cost of Development (c)
1	200	0.6	1	0.85	1	20
2	250	0.7	1	0.85	1.1	25
3	300	0.75	2	0.85	0.6	30
4	280	0.8	1	0.85	1	22

. 5	350	0.85	1	0.85	1.2	35

Table 1. Hypothetical data for development of 5 attributes

Note: The revenue and cost values are given in millions of dollars, while time to development is given in years.

We first consider the case of sequential development and use equation (7) to derive the expected reward for each attribute and equations (19) and (18) respectively to derive the Gittins indices and the optimum reward. The application of Gittins Index strategy results in arm 4 (four product development teams) as the optimum arm, with N* = 5(Please refer to Figure 1). This implies that arm 4 is the optimum arm during the development of all 5 attributes and the corresponding total optimum discounted reward using arm 3 is found to be $357.03 million.

For a second test case, which also assumes sequential development, we use the same data in Table 1; however, it is also assumed that development cost is not necessarily a multiple of number of teams for all attributes. This case is considered to take into account the fact that not all costs will get multiplied with an increase in the number of teams. For example, some of the fixed costs associated with buildings, capital equipment, transportation of raw material and prototypes may not always double when arm 1 is replaced by arm 2. Whereas, depending upon the nature of the attribute, sometimes the cost of employing an extra product development team can be more than doubled, if they have to be hired afresh or temporarily shifted from one development center to the other. Thus, the application of the sequential development model with only change in cost data resulted in arm 3 as the optimum arm for the first 4 attributes (Please refer to Figure 2), while arm 4 becomes the optimum arm for the last attribute. The total discounted reward for this decision rule $A = (a_3, a_3, a_3, a_3, a_4)$, where a_k denotes arm 'k' is found to be $352.9 million.

Finally, we consider the case of overlapped development with the same data in Table 1. However, in this case, we assume that the concurrent development of all attributes occurs during the first 5 periods ($T_c = 5$), whereas the maximum allowed stipulated time is 20 periods ($T = 20$), beyond which the revenue from the new product will reduce to zero. The independent development of all attributes is assumed to begin immediately after the concurrent development and take corresponding times (T_j, j=1,...,5) given in column 5 of Table 1. The rate of development during the concurrent phase is assumed to be equal to 0.6 (λ). For this case, we apply the overlapped development model developed in theorem 2, and calculate the expected reward using equation (13). The Gittins indices are calculated again using equation (19) and the discounted optimum reward is calculated using equation (18). Here again, we find that arm 3 is the optimum arm during the development of all 5 attributes (Please refer to Figure 3). However, the overlapped development seems to have resulted in higher reward this time, with the total optimum discounted reward in this case being equal to $525.82 million.

5. Discussion and Conclusions

In this paper various quantitative models have been developed to arrive at optimal decision rules that enable faster product development, while ensuring a given quality level. We have also shown that these New Product Development models in the multi armed bandit framework are amenable to the use of the Gittins Index strategy, which reduces the search space significantly and hence can assist practitioners in choosing the optimum number of teams, even in case of large number of attributes that usually arise in complex product development scenarios. In particular, the overlapped development model, developed in section 3.4 can be generalized to represent all set based concurrent engineering scenarios of Toyota's product development process, starting from the concept development stage to the production scale-up for the new product. The results of hypothetical examples discussed in the previous section show that, despite the higher cost of parallel teams, the total cumulative reward turns out to be higher when multiple teams are deployed, giving empirical justification to the arguments put forth in the earlier sections.

One of the most important features of this framework at any given stage of development is the ability of MAB to take into account the information gathered in the previous stages and the fact that current and future decisions are made on the basis of information available so far. For example, if we look at the choice of design alternatives (in other words *design flexibility*) enabled by parallel teams in particular, at any given stage, the development of the current attribute and its corresponding reward is closely related to the *flexibility* allowed by the previous attributes, which are already developed. Thus, if more *flexibility* was allowed in the previous stages by the optimum decision rule, it is more likely that the most efficient arm that allows *design flexibility* at an optimum cost and time trade-off was chosen and will continue to be chosen until this optimality breaks, at which point there will be another optimum arm - which may contain lesser number of teams, assuming the cost of multiple development teams will increase exponentially in comparison to the benefits accrued by using them or *vice a versa* - which will take over the development. Thus, the opportunity to use parallel teams that can greatly enhance quality of development at a faster rate and at a lesser cost during the initial stages (or later stages) of development, which in turn significantly increases the expected reward, can be achieved by modeling the parallel product development team problem with a MAB framework and using the Gittins index strategy.

One of the major limitations of this work is the lack of real data on New Product Development projects to test the current models with. Access to real time data and opportunity to apply these models in ongoing New Product Development projects are likely to provide further insights that would be immensely useful for future research in this area. Some of the assumptions, like independence of reward functions across different arms, may not hold good in all product development scenarios. Hence, there is a need to develop models that allow for dependency between the arms. The complexity of the MAB framework and the Gittins decision rule does not allow for derivation of simple analytical conditions between the time to development, cost and rate of development of various attributes, which would have greatly enhanced the understanding of various New Product Development mechanisms.

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Figures

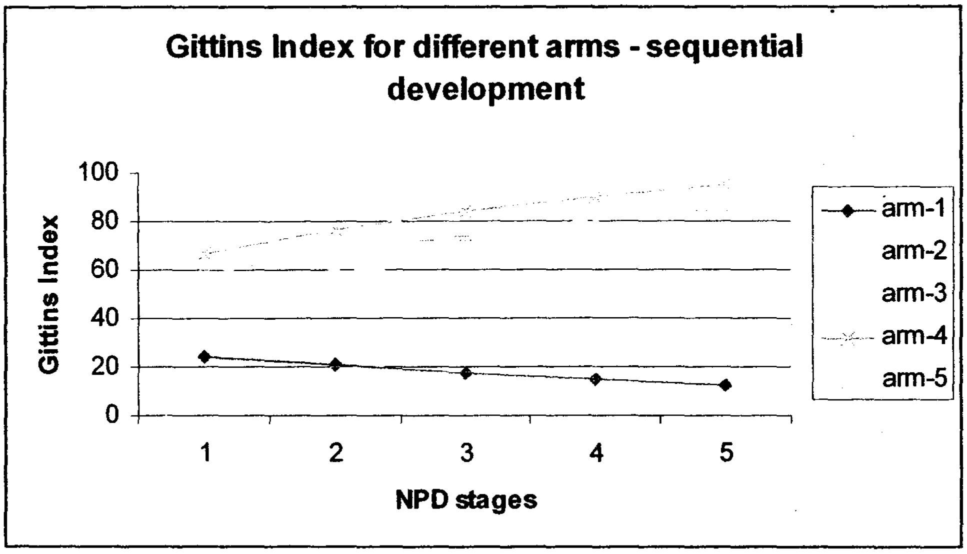

Figure 1. Case 1 of Sequential Development – The Gittins index values for different arms across 5 periods

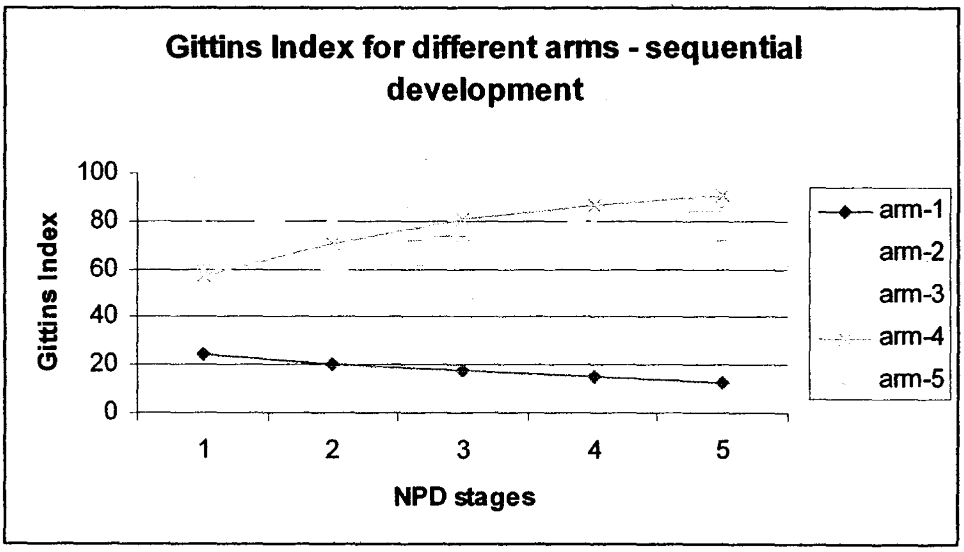

Figure 2. Case 2 of Sequential Development – The Gittins index values for different arms across 5 periods

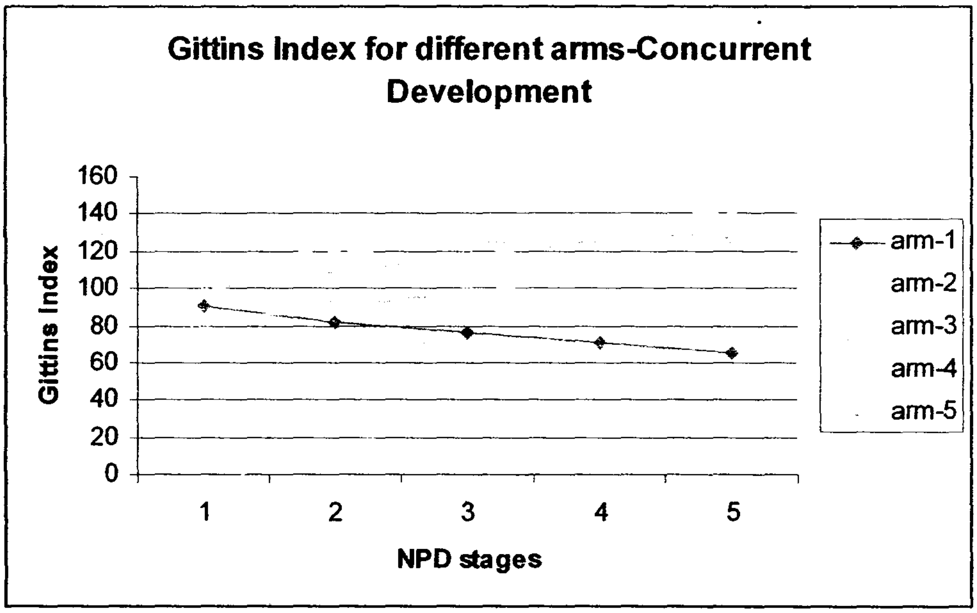

Figure 3. Overlapped Development – The Gittins index values for different arms across 5 periods