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Can Audits Deter Performance Exaggeration?

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Can Audits Deter Performance Exaggeration?

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Abstract

Exaggeration of performance metrics (revenue, product efficacy, ad viewership, etc.) by entrepreneurs to investors and clients is a common problem in high tech entrepreneurship. We model this as a principal-agent problem in a contract-theoretic setting, where the entrepreneur (agent) can undertake costly actions to strategically lie about a key performance metric (agent type) in order to extract higher payment from the investor (principal). We demonstrate that the optimal contract features widespread exaggeration by all entrepreneur types, and the investor exploits it as a screening mechanism to ordinally rank the entrepreneur by his true underlying type. We study the effect of an audit in which, if caught cheating, the agent pays a penalty. We show that rather than deterring fraud, audits actually amplify the degree of exaggeration.

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"Every tech startup exaggerates to get funding."— Sachin Dev Duggal, founder and ousted "Chief Wizard" of artificial intelligence startup Engineer.ai (Purnell and Olson, 2019)

1 Introduction

The above quote by a prominent entrepreneur alludes to a problem that may be the entrepreneurship world's worst-kept secret, that a lot of startup founders egregiously exaggerate key performance metrics (revenue, product efficacy, daily active users, ad viewership metrics, etc.) to boost their valuation for funding or takeovers, or simply command higher prices when dealing with potential clients for future business. Sachin Dev Duggal was the co-founder of artificial intelligence startup Engineer.ai, which purportedly allows developers to build their own software applications using an easy-to-use graphical user interface. Though the company claimed that it used artificial intelligence at the back-end, several former employees revealed that its capabilities were greatly exaggerated by Duggal, and the company was actually using human engineers in India to do the job. Thus, the company's venture capital funding of $29.5 million may have been based on false premises (Purnell and Olson, 2019).

Theranos and its founder Elizabeth Holmes constitute another case in point. The medical diagnostics startup (valued at $9 billion as late as 2017) was found to have fraudulently exaggerated the accuracy of its in-home blood testing kits. Further investigations following up on claims by an internal whistle-blower, revealed falsification of tests, suppression of quality control concerns, and intimidation of whistle-blowers by the startup's top management. Holmes faces criminal cases for her actions that may have jeopardized not only Theranos's investors, but also patients using its diagnostic kits (Carreyrou, 2016; LA Times, 2019).

Revenue exaggeration by entrepreneurs, usually before a round of funding, an initial public offering, or takeover, is another major source of concern in the startup world. Employees of vegan food startup Hampton Creek have been caught buying their own products from supermarket shelves, attempting to make their product seem more popular to retail partners (Griffith, 2016). Twitter acquired ad tech company MoPub in 2013 when its founder boasted of annual revenues approaching $100 million on his blog, only to see revenues of $6.5 million in the first two quarters of that year (Popper, 2015). Shopping deals startup Groupon recalculated its financial results for the previous three years "to correct for an error" in its revenue reporting, leading to more than halving of its claimed revenue from $1.52 billion to $688 million in the first half of 2011 (De La Merced and Rusli, 2011). Chinese e-commerce venture Pinduoduo stands accused by investment firm

Blue Orca, for allegedly exaggerating its gross revenue before its initial public offering (Nikkei Asian Review, 2018).

Some new (and occasionally even mature) ventures fraudulently tweak other key performance metrics to entice their clients or partners to either do business with them, or command higher prices for their services. Ride sharing platform Uber was fined $20 million by the US Federal Trade Commission for exaggerating driver earnings (FTC, 2017). Consulting firm TPI finds that outsourcing reduces costs by an average of 15% as compared to vendors' claims of up to 60% (Price, 2007).

In a recent high profile case, Facebook agreed to pay damages of $40 million in an out-of-court settlement to content providers after admitting that its video viewership numbers were inflated by up to 900% (Morris, 2019). Facebook's egregious video metrics inflation has consequences beyond just the $40 million fine that the social media giant paid. A viral series of tweets[1] by comedian and television actor Adam Conover outlines how College Humor, a digital video content creator and his previous employer, switched platforms from YouTube to native video hosting on Facebook due to the latter's inflated claims of video viewership. This allegedly resulted in massive falls in viewership numbers, and hence advertising on College Humor, jeopardizing its profitability. Conover claims that Funny or Die, another well-known video content provider, also suffered similar setbacks as College Humor. Note that apart from the losses incurred by these portals, there were competitive consequences for YouTube, Facebook's main competitor for such video hosting (and College Humor's previous native host). In this case however, metrics falsification by an established company may have harmed its clients who are new ventures.

In this paper, we develop a model in which an entrepreneur seeks payment from an investor, based on a key performance metric. The entrepreneur can undertake a costly action to exaggerate his performance metric and cover up his lie, but under the threat of being caught by an imperfect and non-strategic audit. Our model builds on Crocker and Morgan (1998) who model inflation of losses by insurance claimants and of Crocker and Slemrod (2007), who model strategic misreporting of earnings in financial reports by CEOs. However, they assume that such claims are unverifiable. In our approach, we relax this assumption and and introduce a non-strategic audit in the game, which can detect fraud with a given probability. On successful fraud detection, the entrepreneur experiences escalation of his costs, over and above what is incurred in falsifying the metric. An exogenous fraction of this extra cost is then paid to the investor as restitution, while the remaining is either paid as fines to a regulatory body, or incurred as reputation cost by the entrepreneur.

[1] https://twitter.com/adamconover/status/1183209875859333120

Using a contract-theoretic approach coupled with optimal control theory, we demonstrate that the optimal contract must accommodate widespread exaggeration by entrepreneurs. We further demonstrate that the amount of falsification in the reported performance metric in fact increases with the true underlying metric of the entrepreneur, i.e. his type. We postulate a signaling mechanism as the cause of this phenomenon. This is because the only way for an entrepreneur to credibly signal his true underlying metric (type) is to exaggeratedly report it, because other types are doing the same. The investor must tolerate this, and even compensate the entrepreneur for the optimal faking level, because he can exploit this as a screening mechanism to ordinally sort the entrepreneur by his true underlying metric.

Our model further demonstrates that the audit backfires, i.e. faking level by the entrepreneur increases, and investor's profit decreases as the exogenous accuracy of the non-strategic audit increases. Furthermore, an increase in the penalty on detection, i.e. the exogenous cost escalation factor, also encourages more faking while simultaneously reducing the investor's profit under the same conditions. We posit that this is because the audit distorts the underlying signaling process, and the cost penalties on detection are not sufficient to deter falsification. We note similarities of our results with manufacturer-supplier audits in Plambeck and Taylor (2015).

The contribution of this paper is threefold. First, it explains a well-known scenario in the world of high tech entrepreneurship. The type-signaling mechanism we show at the heart of the attribute exaggeration problem explains why this phenomenon is so widespread. Second, we contribute to the literature on backfiring audits. Our observations in this context are significant for both potential investors and other principals like business clients, because intuition suggests that close auditing should deter fraud. Indeed, there are parallel efforts ongoing to develop better fraud detection mechanisms (for human auditors) and algorithms (for automated audits). While audits may work in many scenarios to deter fraud, our model indicates that more accurate audits actually distort the signaling mechanism underlying the principal-agent game here. Third, our model can be used to advise regulators and policy makers on appropriate penalties to impose on fraudulent entrepreneurs. Escalating the penalty on the agent actually encourages fraud, and thus, design of punitive measures on fraud detection may be rethought.

The fraudulent exaggerations cited above are not mere puffery (usage of unverifiable claims like "this tie looks great on you" or "this car drives very well"), a common tactic in consumer advertising, and legally recognized as a class of advertising claims that the law assumes are "incapable of causing any consumer deception" (Preston, 1977; Rotfeld and Rotzoll, 1980). Note that in each of these examples, no verifiable metric is used to define "great" or "drives well."[2] Puffery is beyond the scope of our work.

We model an entrepreneur who undertakes costly actions to fraudulently misreport a key performance metric used by an investor, that is both *measurable* and *verifiable*. In the presence of complete and unbiased information to both principal and agent, it would not be possible for the agent to exaggerate metrics like revenue, blood test accuracy, video viewership or Uber driver earnings. However, in many cases (including all of the above), the entrepreneur has access to company databases which he can tamper with, pliant employees whom he can collude with, potential whistle-blowers whom he can threaten, and click farms he can employ to astro-turf online engagement, and thus falsify information leading to exaggerated performance metrics. Such actions also make it difficult for regulators and auditors to detect fraud. While many high-profile cases of fraud are both detected and punished, it is clear that several are not. In fact, informal chats with startup founders reveal the truth behind Sachin Dev Duggal's statement at the beginning of this paper; that several founders do exaggerate key performance metrics to both investors and clients to further their business interests. Our research aims to uncover the incentive systems leading to such fraudulent exaggeration of performance metrics under imperfect audit.

The rest of this paper is organized as follows. In section 2, we talk about audits and their role as deterrents of misdemeanor in various contexts. In section 3, we provide some methodological background, before specifying the principal-agent problem in a contract-theoretic framework. We solve for the optimal contract using optimal control theory, and discuss comparative statics to investigate the effects of audit on fraud and investor welfare. Finally, we offer concluding remarks in section 4.

2 Audits as deterrents of misdemeanor

Intuition suggests that misdemeanor in general, and fraud in particular, can be dissuaded by close auditing, and appropriate penalties imposed in case a fraud is exposed. These penalties could include damages paid to the affected principal, or fines to non-strategic regulators like the Federal Trade Commission or the Securities and Exchange Commission. Intuition may further suggest that if a regulatory body imposes a large fine, it is only fair that the principal is given that amount (after subtracting any expenses the regulator may have) in the in the interest of compensating him for losses incurred. However, there exists a vast literature on audits, with mixed results, based on the kind of fraud being studied. In the next two paragraphs, we summarize a few studies germane to our problem

[2] Though these claims may not not be credible individually, Chakraborty and Harbaugh (2014) analytically show that puffery can still be perceived as credible, because exaggerating one aspect of a product comes at the implicit cost of not talking about another aspect.

at hand.

In a study of click fraud, Wilbur and Zhu (2009) demonstrate how a third party auditor can benefit the search industry. In an empirical study, Dionne et al. (2009) find some deterrence effects of an optimal policy involving special investigation of "red flags" while processing insurance claims. Hoopes et al. (2012) find that increasing the probability of internal revenue service audits on US companies has a positive effect on their cash effective tax rates.

Along with the literature in support of audits, there exists a body of literature finding its limits. An early example is Townsend (1979), who demonstrates that for insurance audits with deterministic auditing, there exists a cutoff above which auditing claims is sub-optimal for the principal. More recently, Plambeck and Taylor (2015) find conditions for a "backfiring" audit, where increasing auditing effort by a manufacturer (principal) over a supplier (agent—who may mistreat workers, cause pollution etc.) and imposing higher penalties to an offending supplier, makes the principal worse off. This is because the manufacturer expends more effort in evading the audit rather than avoiding the transgression in question. Rather, they show that squeezing the supplier's margins in the "backfiring" condition is a better deterrent for the transgressions. Our study, though methodologically different, and modeling a different kind of transgression, finds a "backfiring" audit condition as well, dictated by the characteristics of the cost function. Boleslavsky et al. (2017) also find the prevalence of exaggeration by entrepreneurs competing for scarce investor capital, and argue that investors are worse off with more transparency. In a model of social media influencers faking their follower counts to online advertisers, Anand et al. (2019) find that faking levels are unaffected, but the advertiser's payoff decreases with audit accuracy.

A large body of research, both academic and applied, focuses on better fraud detection methods and technologies under different scenarios (e.g. Bolton and Hand, 2002). However, as we demonstrate in our case, there exist scenarios where more accurate detection can actually backfire for the principal. In our case, many commonly assumed cost functions (including quadratic cost) lead to backfiring. Here, increasing audit accuracy or penalty imposed for fraud leads to an increase in faking levels, as well as a decrease in the principal's profit. In our model, the fraud itself is a signaling mechanism by which the principal can screen the agent based on its underlying type (true metric). Introducing audits distorts incentives, increases fraud, and harms the principal. Thus, under such circumstances, our model does not recommend auditing exaggerated claims of performance metrics by entrepreneurs.

3 Model development

3.1 Methodological background

We adopt a contract-theoretic approach, where the agent can exaggerate its privately known performance metric, i.e. its underlying type, to extract higher payment from the principal. Such fraud is analogous to inflated insurance claims as modeled by Crocker and Morgan (1998). However they assume that the agent's claim is unverifiable and demonstrate how an optimal contract must tolerate some degree of misrepresentation by the agent. Crocker and Slemrod (2007) extend this model to CEOs reporting exaggerated earnings to shareholders in annual reports.

Unlike Crocker and Morgan (1998), we do not assume that the agent can always obfuscate performance metrics to evade auditors. As the examples in section 1 illustrate, even the most meticulously planned fraud occasionally unravels. Thus, we develop our model under the assumption that audits are imperfect but not completely ineffective, and can detect fraud in the agent's claims with a fixed probability. If the agent commits fraud, he incurs a cost that rises progressively with the degree of misrepresentation of the performance metric. If the agent does get caught, he incurs further costs, part of which is paid back to the affected principal. The remaining penalties incurred could be fines paid to regulatory bodies, loss of reputation, suspension of business, etc.

In section 3.2, we develop an optimal contract between a principal and agent, where the latter can indulge in costly falsification of his type, with the possibility of getting caught by a non-strategic third party audit. We frame our problem as one of optimal control and derive its solution, followed by comparative statics in section 3.4 and an illustration in section 3.5.

3.2 Model specification

We now outline the model in detail. Table 1 serves as a reference summarizing algebraic notation used henceforth.

Insert table 1 about here

Consider a risk-neutral principal (investor or client) and a risk neutral agent (entrepreneur). The entrepreneur's value is judged by its performance metric m (revenue, video viewership, driver earnings, etc) which we assume as continuous: the higher the value of the entrepreneur's metric, the higher is the subjective value expected by the investor, denoted by $A(m)$ where $A'(m) > 0$. The metric m is private information to the entrepreneur, who reports a possibly inflated value $u(m) \geq m$. The investor observes only the reported performance metric $u(m)$ and not its underlying true value m. The reporting function $u(m) = m$ indicates no falsification, while $u(m) > m$ indicates falsification by the entrepreneur. The investor is aware that the true metric m is distributed in $[m_L, m_H]$ according to the probability density function $f(m)$.

Falsification of the metric m imposes a cost denoted by $c(u(m) - m)$ on the entrepreneur. This cost includes not just the cost of faking numbers, but also cover-up costs, say bribing regulators, silencing whistle-blowers, etc. We assume that (a) this cost is increasing in the degree of falsification, i.e. $c'(z) > 0 \; \forall z > 0$, (b) that falsification is progressively more expensive i.e. $c''(z) > 0 \; \forall z \geq 0$, (c) no falsification entails no cost, i.e. $c(0) = 0$ and (d) that the minimum cost is for no falsification, i.e. $c'(0) = 0$.

The entrepreneur is paid by the investor based on his performance metric. While one would expect this payment to be a function of the reported metric $u(m)$, we invoke the revelation principle instead to look for only direct mechanisms where the entrepreneur's compensation $v(m)$ is a function of its true metric m (Myerson, 1979). The principal-agent game unfolds in the presence of a non-strategic third party audit, where the agent's fraud may be detected with an exogenous probability γ. We assume that the audit never flags a non-fraudulent entrepreneur. If the entrepreneur is caught indulging in fraud, his cost is multiplied by an exogenous factor $\delta > 1$. Of this total cost $\delta c(u(m) - m)$, the component $c(u(m) - m)$ is already incurred in falsification, and of the remaining $(\delta - 1)c(u(m) - m)$, an exogenous fraction θ is returned to the investor as restitution. The remaining fraction $1 - \theta$ could either be fines to regulatory bodies, or reputation costs to the entrepreneur. Figure 1 illustrates this.

Insert figure 1 about here

The principal's profit can now be written as,

$$\begin{aligned} \Pi(u(m), v(m)) &= A(m) - (1-\gamma)v(m) + \gamma\{\theta(\delta - 1)c(u(m) - m) - v(m)\} \\ &= A(m) - v(m) + \gamma\theta(\delta - 1)c(u(m) - m) \end{aligned} \quad (1)$$

with the principal's objective being to maximize expected profit, expressed as,

$$\max_{u(m), v(m)} \int_{m_L}^{m_H} \Pi(u(m), v(m)) f(m) dm \quad (2)$$

The agent's payoff function is expressed as,

$$\begin{aligned} Y(u(m), v(m), m) &= (1-\gamma)(v(m) - c(u(m) - m)) + \gamma(v(m) - \delta c(u(m) - m)) \\ &= v(m) - (1 - \gamma + \gamma\delta)c(u(m) - m) \end{aligned} \quad (3)$$

Note that the agent's payoff function is independent of the fraction θ which is apportioned between to the principal. Incentive compatibility for the agent dictates that,

$$Y(v^*(m), u^*(m), m) \geq Y(v^*(\hat{m}), u^*(\hat{m}), m) \qquad \forall \hat{m} \neq m \in [m_L, m_H] \tag{4}$$

Furthermore, an optimal contract must satisfy the individual rationality constraint, such that the agent's payoff must exceed his outside option. We normalize this outside option to zero, and thus have,

$$Y(u(m), v(m), m) \geq 0 \tag{5}$$

The principal's optimization program is thus objective function (2) subject to the incentive compatibility and individual rationality constraints of equation (4) and (5) respectively. Note that the metric m is exogenous and *not* a decision variable that can be chosen during optimization. The optimization problem is rather to choose the functions $u(m)$ and $v(m)$, turning this into an optimal control problem where Y is the state variable, with its equation of motion represented as,

$$\frac{dY}{dm} = \frac{\partial Y}{\partial m} \tag{6}$$

3.3 Optimal contract

We can now express the principal's optimization problem with the following Hamiltonian,

$$\mathbb{H} = \Pi(v, u) f(m) + \lambda(m) Y_m + \mu Y(v, u, m) \tag{7}$$

where $\lambda(m)$ is the co-state variable, $u(m)$ is the control variable and μ is a Lagrange multiplier. Using the Pontryagin maximum principle, Crocker and Morgan (1998) derive the two following necessary conditions of optimality,

$$f \cdot (\Pi_u - \Pi_v Y_u / Y_v) + \lambda (Y_{um} - Y_{vm} Y_u / Y_v) = 0 \tag{8}$$
$$\frac{d\lambda}{dm} = -f \cdot \frac{\Pi_v}{Y_v} - \lambda \frac{Y_{vm}}{Y_v} - \mu \tag{9}$$

The above conditions lead to the following proposition,

Proposition 1. ***(Optimal contract)*** *The optimal contract is characterized by:*

$$\frac{F(m)}{f(m)} = \left(\frac{c'(u(m) - m)}{c''(u(m) - m)} \right) \cdot \left(\frac{1 + \gamma(\delta - 1)(1 - \theta)}{1 - \gamma + \gamma\delta} \right) \tag{10}$$
$$v(m) = (1 - \gamma + \gamma\delta) \cdot \left(c(u(m) - m) + \int_{m_L}^{m} c'(u(t) - t) dt \right) \tag{11}$$

Proof. See appendix A.1 □

Corollary 1.1. ***(Every type except m_L is faking)*** *The following hold true for the optimal contract:*

$$u(m) > m \quad \forall m > m_L \tag{12}$$
$$u(m_L) = m_L \tag{13}$$

Proof. See appendix A.2 □

The above proposition indicates that all agent types except the lowest (with $m = m_L$), over-report their metric. The logic behind this is that the faking acts as a type-signaling mechanism. Given the costs of faking and the incentive structure in place, the agent must over-report his performance metric to convey his true underlying metric, i.e. type. This optimal level of faking $u(m)$ is determined by equation (10). The optimal contract dictates the principal to compensate the agent for this fraud, as indicated in equation (11). While the above scenario may seem to be a completely losing proposition for the principal, he can actually use equation (10) as a screening mechanism to ordinally rank agents by their true underlying metric m. We illustrate this with an example in section 3.5.

Another interesting consequence of this setup, which our illustration shows, is the possibility of several types actually reporting a metric that is beyond m_H—the upper limit of the support—and hence very obviously false. Such results have also been observed in other contexts (e.g. Maggi and Rodriguez-Clare, 1995; Crocker and Morgan, 1998; Anand et al., 2019). The principal also knows this, and tolerates it because of the bijective mapping between m and $u(m)$.

Implementability and sufficiency

Implementability requires,

$$\frac{\partial}{\partial m}\left(\frac{Y_u}{Y_v}\right) \cdot \frac{du}{dm} > 0 \tag{14}$$

Since $Y_{um} = (1 - \gamma + \gamma\delta)c''(\cdot)$ and $Y_v = 1$, the term $\partial(Y_u/Y_v)/\partial m$ becomes $c''(u(m) - m) \cdot (1 - \gamma + \gamma\delta) > 0$. Thus $u' > 0$ is necessary for implementability. $Y_m > 0$ implies sufficiency, i.e. the participation constraint binds only at $m = m_L$ leading to $Y(m_L) = 0$; and $Y(m) > 0 \ \ \forall m > m_L$. This is ensured if $c' > 0$.

3.4 Effects of audit

With the optimal contract derived in proposition 1, we now investigate the effects γ, δ and θ on the optimal level of exaggeration. Proposition 2 establishes how the audit amplifies exaggeration.

Proposition 2. ***(Effects of audit parameters on faking)*** *The following hold true* $\forall m > m_L$*:*

$$\begin{aligned} \partial u/\partial\gamma &> 0 && (15)\\ \partial u/\partial\delta &> 0 && (16)\\ \partial u/\partial\theta &> 0 && (17)\end{aligned}$$

Proof. See appendix B.1 □

Proposition 2 illustrates how the audit distorts the signaling mechanism. It intensifies exaggeration because the cost structure necessitates higher levels of exaggeration for the agent to credibly signal his true type. In turn, the agent passes on the higher costs of more exaggeration to the principal, as indicated in corollary 2.1. Further we show how an audit may reduce the principal's payoff in proposition 3.

Corollary 2.1. ***(Effects of audit parameters on agent's payment)*** *The following hold true* $\forall m > m_L$*:*

$$\begin{aligned} \partial v/\partial\gamma &> 0 && (18)\\ \partial v/\partial\delta &> 0 && (19)\\ \partial v/\partial\theta &> 0 && (20)\end{aligned}$$

Proof. See appendix B.2 □

Proposition 3. ***(Effects of audit parameters on principal's payoff)*** *The following hold true* $\forall m > m_L$*:*

$$\begin{aligned} \partial \Pi/\partial\gamma &< 0 && (21)\\ \partial \Pi/\partial\delta &< 0 && (22)\end{aligned}$$

Proof. See appendix C □

3.5 Illustration

We now present a brief illustration of our model using a uniform distribution with support $[0, 100]$ and a quadratic cost function. The uniform distribution is specified as $\mathbb{U}(m_L, m_H)$, such that $f(m) = 1/(m_H - m_L)$ and $F(m) = (m - m_L)/(m_H - m_L)$.

The commonly used quadratic function $c(z) = az^2$; $a > 0$ satisfies all the assumptions of section 3.2. From equation (10) we have,

$$\begin{aligned}
\frac{F(m)}{f(m)} &= \left(\frac{c'(u(m)-m)}{c''(u(m)-m)}\right) \cdot \left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right) \\
\Rightarrow m - m_L &= (u(m)-m)\left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right) \\
\Rightarrow u(m) - m &= \frac{(1-\gamma+\gamma\delta)}{(1+\gamma(\delta-1)(1-\theta))}(m - m_L)
\end{aligned}$$

The above expression for $u(m) - m$, i.e. the level of falsification, is thus a function of $m, \gamma, \delta, \theta$. Figures 2 - 4 illustrate how the level of falsification increases with each parameter, *ceteris paribus*.

Insert figures 2 - 4 continuously about here

4 Conclusion

We model the dynamics of costly exaggeration by entrepreneurs (agents) of a key performance metric (agent type) to a potential investor or client (principal), by developing an optimal contract between the two. We find that all agent types (except the lowest type) find it worthwhile to exaggerate their performance metric, and the optimal reported metric acts as a signaling mechanism to indicate their true underlying type. Interestingly, we find that the principal is better off without imperfect audits under reasonable assumptions on the falsification cost. Thus, given that audits only increase exaggeration, we recommend that regulators avoid this practice, however counter-intuitive it may seem. This is because the audit interferes with the signaling mechanism, and any increase in either audit accuracy or punitive penalties, actually results in making the investor (whom regulators like the Securities and Exchange Commission or Federal Trade Commission must protect) worse off.

Our results thus lend credence to the cynical Silicon Valley aphorism of "fake it till you make it" with a sound economic explanation. While we find that exaggeration of key performance metrics by entrepreneurs is unavoidable, this phenomenon does have a silver lining for the investor. He can exploit the bijective mapping between the actual metric m and reported metric $u(m)$ to ordinally sort the entrepreneur by his underlying type m.

Our model lends itself to an interesting extension, which we suggest as a promising avenue for future research. It accounts for misreporting of only one performance metric. In reality, some entrepreneurs may exaggerate multiple attributes. For example, Theranos's Elizabeth Holmes stands accused of exaggerating not only medical test accuracy, but also of exaggerating revenue by a factor of 1,000 (Aiello, 2018). An investigation of such multi-attribute exaggeration, using the approach of Frankel and Kartik (2019) is possible.

In conclusion, we model an undesirable but all-pervasive scenario in the world of entrepreneurship—that of performance metric exaggeration—and demonstrate how audits meant to curtail this may backfire. Our findings are important to investors, policy makers and entrepreneurship researchers.

Appendices

A Proof of proposition 1 and corollary 1.1

A.1 Proof of proposition 1

We note that the participation constraint in equation (5) is slack ($Y > 0$) and hence the corresponding Lagrange multiplier μ in equation (9) is set to zero. Using $\mu = 0$ in equation (9), we obtain:

$$\frac{d\lambda}{dm} = f(m) \tag{A1}$$

which along with the transversality condition $\lambda(m_L) = 0$ yields:

$$\lambda(m) = F(m) \tag{A2}$$

where $F(m) = \int_{m_L}^{m} f(t)dt$. Substituting the value of λ in equation (8) we obtain

$$\frac{F(m)}{f(m)} = \left(\frac{c'(u(m)-m)}{c''(u(m)-m)}\right) \cdot \left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right)$$

Now to derive $v(m)$ we note that,

$$\int_0^Y dY = \int_{m_L}^{m} \frac{dY}{dm} dm = \int_{m_L}^{m} \frac{\partial Y}{\partial m} dm$$

$$Y = \int_{m_L}^{m} (1-\gamma+\gamma\delta) \cdot c'(u(t)-t)dt \tag{A3}$$

$$v(m) = (1-\gamma+\gamma\delta)\left[c(u(m)-m) + \int_{m_L}^{m} c'(u(t)-t)dt\right] \tag{A4}$$

A.2 Proof of corollary 1.1

We have now established equation (10), i.e.,

$$\frac{F(m)}{f(m)} = \left(\frac{c'(u(m)-m)}{c''(u(m)-m)}\right) \cdot \left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right)$$

For all $m > m_L$, it is evident that the left hand side of the above equation is positive. We know that $c'' > 0$ by assumption and the term $\left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right)$ is also positive. Thus it must be that $c'(u(m) - m) > 0$ which is only possible if $u(m) - m > 0$. Thus $u(m) > m \; \forall m > m_L$, which is condition (12).

By a similar logic, at $m = m_L$, the left hand side of equation (10) is zero which is only possible if $u(m_L) - m_L = 0$ or $u(m_L) = m_L$, which is condition (13).

B Proof of proposition 2 and corollary 2.1

B.1 Proof of proposition 2

First of all, we look at equation (10), defining the left (and right) hand side as Z:

$$Z \equiv \frac{F(m)}{f(m)} = \left(\frac{c'(u(m)-m)}{c''(u(m)-m)}\right) \cdot \left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right)$$

Thus,

$$\begin{aligned} \frac{dZ}{d\gamma} &= 0 = \frac{\partial Z}{\partial \gamma} + \frac{\partial Z}{\partial u}\frac{\partial u}{\partial \gamma} \\ \Rightarrow \frac{\partial u}{\partial \gamma} &= -\frac{\partial Z/\partial \gamma}{\partial Z/\partial u} \end{aligned} \tag{A5}$$

Similarly,

$$\frac{\partial u}{\partial \delta} = -\frac{\partial Z/\partial \delta}{\partial Z/\partial u} \tag{A6}$$

$$\frac{\partial u}{\partial \theta} = -\frac{\partial Z/\partial \theta}{\partial Z/\partial u} \tag{A7}$$

Now, partially differentiating the right hand side of equation (10) with respect to γ, δ and θ respectively, we have

$$\frac{\partial Z}{\partial \gamma} = \frac{-(\delta-1)\theta}{(1-\gamma+\gamma\delta)^2}\left(\frac{c'}{c''}\right) < 0 \tag{A8}$$

$$\frac{\partial Z}{\partial \delta} = \frac{-\gamma\theta}{(1-\gamma+\gamma\delta)^2}\left(\frac{c'}{c''}\right) < 0 \tag{A9}$$

$$\frac{\partial Z}{\partial \theta} = \frac{-\gamma(\delta-1)}{(1-\gamma+\gamma\delta)}\left(\frac{c'}{c''}\right) < 0 \tag{A10}$$

Partially differentiating the right hand side of equation (10) with respect to u, we have

$$\frac{\partial Z}{\partial u} = \left(\frac{1+\gamma(\delta-1)(1-\theta)}{1-\gamma+\gamma\delta}\right) \cdot \left(\frac{(c'')^2 - c'c'''}{(c'')^2}\right) \tag{A11}$$

From equation (A11), (A8) and (A5), it is evident that $\partial u/\partial \gamma > 0$ iff $(c'')^2 - c'c''' > 0$. It is easy to verify that any function that does not satisfy $(c'')^2 - c'c''' > 0$ is incompatible with the assumptions of section 3.2. This is because $c'(0) = 0$ and $c''(0) > 0$. At 0, we have $(c'')^2 - c'c''' = (c'')^2 > 0$.

Similar logic holds for $\partial u/\partial\delta$ and $\partial u/\partial\theta$.

B.2 Proof of corollary 2.1

The expression for $v(m)$ as given by equation (11) is,

$$v(m) = (1-\gamma+\gamma\delta)\cdot\left(c(u(m)-m)+\int_{m_L}^{m} c'(u(t)-t)dt\right)$$

We note that $1-\gamma+\gamma\delta$ is an increasing function of both γ and δ. Furthermore, from proposition 2, $u(\cdot)$ is clearly an increasing function of γ, δ and θ. Also, given that $c' > 0$ and $c'' > 0$, it is evident that both $c(\cdot)$ and $c'(\cdot)$ increase with γ, δ and θ. Therefore, it is trivially true that $v(m)$ also increases with each of these parameters.

Nevertheless, given that we require formal expressions of $\partial v/\partial\gamma$ and $\partial v/\partial\delta$ to prove proposition 3 (see appendix C), we will still derive these and prove explicitly that these are negative. We use the Leibniz integral rule to partially differentiate equation (11) with respect to γ to get,

$$\begin{aligned}\frac{\partial v(m)}{\partial\gamma} &= (\delta-1)\left(c(u(m)-m)+\int_{m_L}^{m} c'(u(t)-t)dt\right)\\ &+(1-\gamma+\gamma\delta)\left(c'(u(m)-m)\frac{\partial u(m)}{\partial\gamma}+\int_{m_L}^{m} c''(u(t)-t)\frac{\partial u(t)}{\partial\gamma}dt\right)\end{aligned} \tag{A12}$$

It is easy to verify that the above is positive, because $\partial u(\cdot)/\partial\gamma$ is positive. Similarly we partially differentiate equation (11) with respect to δ to get,

$$\begin{aligned}\frac{\partial v(m)}{\partial\delta} &= \gamma\left(c(u(m)-m)+\int_{m_L}^{m} c'(u(t)-t)dt\right)\\ &+(1-\gamma+\gamma\delta)\left(c'(u(m)-m)\frac{\partial u(m)}{\partial\delta}+\int_{m_L}^{m} c''(u(t)-t)\frac{\partial u(t)}{\partial\delta}dt\right)\end{aligned} \tag{A13}$$

which is also positive, because $\partial u(\cdot)/\partial\delta$ is positive. Finally we partially differentiate equation (11) with respect to θ to get,

$$\frac{\partial v(m)}{\partial\theta} = (1-\gamma+\gamma\delta)\left(c'(u(m)-m)\frac{\partial u(m)}{\partial\theta}+\int_{m_L}^{m} c''(u(t)-t)\frac{\partial u(t)}{\partial\theta}dt\right) \tag{A14}$$

which is positive because $\partial u(\cdot)/\partial\theta$ is positive.

C Proof of proposition 3

We recall equation (1), the principal's payoff, which is,

$$\Pi = A(m) - v(m) + \gamma\theta(\delta - 1)c(u(m) - m)$$

Partially differentiating the above with respect to γ, we have,

$$\frac{\partial \Pi}{\partial \gamma} = -\frac{\partial v(m)}{\partial \gamma} + \theta(\delta - 1)c(u(m) - m) + \gamma\theta(\delta - 1)c'(u(m) - m)\frac{\partial u(m)}{\partial \gamma}$$

Substituting equation (A12) above and simplifying, we have,

$$\begin{aligned}
\frac{\partial \Pi}{\partial \gamma} &= -(1-\theta)(\delta - 1)c(u(m) - m) \\
&\quad - \{(1-\gamma) + \gamma\delta(1-\theta) + \gamma\theta\}\, c'(u(m) - m)\frac{\partial u(m)}{\partial \gamma} \\
&\quad -(\delta - 1)\int_{m_L}^{m} c'(u(t) - t)dt \\
&\quad -(1 - \gamma + \gamma\delta)\int_{m_L}^{m} c''(u(t) - t)\frac{\partial u(t)}{\partial \gamma}dt
\end{aligned} \tag{A15}$$

It is easy to see that the above expression is negative because $\partial u(\cdot)/\partial\gamma > 0$.

We now partially differentiate equation (1) with respect to δ and repeat the above exercise by substituting equation (A13) to obtain,

$$\begin{aligned}
\frac{\partial \Pi}{\partial \delta} &= -\frac{\partial v(m)}{\partial \delta} + \gamma\theta c(u(m) - m) + \gamma\theta(\delta - 1)c'(u(m) - m)\frac{\partial u(m)}{\partial \delta} \\
&= -\gamma(1-\theta)c(u(m) - m) \\
&\quad - \{(1-\gamma) + \gamma\delta(1-\theta) + \gamma\theta\}\, c'(u(m) - m)\frac{\partial u(m)}{\partial \delta} \\
&\quad -\gamma\int_{m_L}^{m} c'(u(t) - t)dt \\
&\quad -(1 - \gamma + \gamma\delta)\int_{m_L}^{m} c''(u(t) - t)\frac{\partial u(t)}{\partial \delta}dt
\end{aligned} \tag{A16}$$

It is easy to see that the above expression too is negative because $\partial u(\cdot)/\partial\delta > 0$.

Why we do not include $\partial\Pi/\partial\theta$ in proposition 3

We partially differentiate equation (1) with respect to θ and repeat the above exercise by substituting equation (A14) to obtain,

$$\begin{aligned}
\frac{\partial \Pi}{\partial \theta} &= -\frac{\partial v(m)}{\partial \theta} + \gamma(\delta-1)c(u(m)-m) + \gamma\theta(\delta-1)c'(u(m)-m)\frac{\partial u(m)}{\partial \theta} \\
&= \gamma(\delta-1)c(u(m)-m) \\
&\quad -\{(1-\gamma)+\gamma\delta(1-\theta)+\gamma\theta\}\, c'(u(m)-m)\frac{\partial u(m)}{\partial \theta} \\
&\quad -(1-\gamma+\gamma\delta)\int_{m_L}^{m} c''(u(t)-t)\frac{\partial u(t)}{\partial \theta}dt
\end{aligned} \tag{A17}$$

The above expression is negative if and only if the right hand side is negative. This imposes some extra constraints on the parameters and/or the cost function, but it is not very intuitive.

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Term	*Description*
m	True performance metric (agent type)
$f(m), F(m)$	Probability density and cumulative distribution function of m
$[m_L, m_H]$	Support of the probability density function f
$A(m)$	Principal's subjective valuation of agent based on m
$u(m)$	Agent's reported metric (type)
$v(m)$	Payment to the agent
$c(u(m) - m)$	Cost of falsification
$\Pi(\cdot)$	Principal's payoff
$Y(\cdot)$	Agent's payoff
$\mathbb{H}$	Hamiltonian
$\lambda(m)$	Co-state variable
μ	Lagrange multiplier
γ	Probability of successful detection of agent's fraud
δ	Penalty factor
θ	Fraction of agent's incurred penalty going to principal

Table 1: Model notation

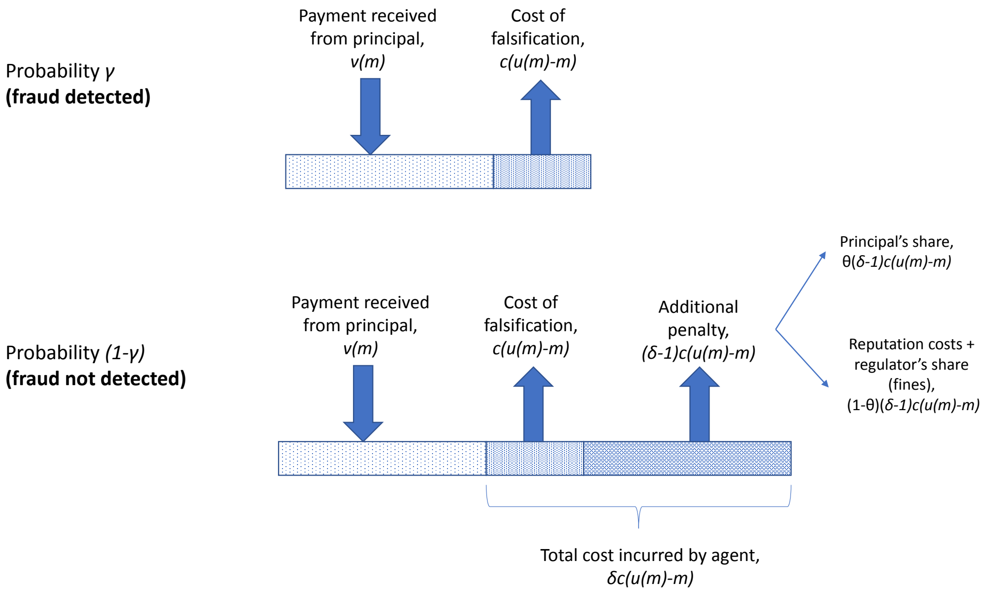

Figure 1: An illustration of the agent's payoffs and costs when not caught and when caught

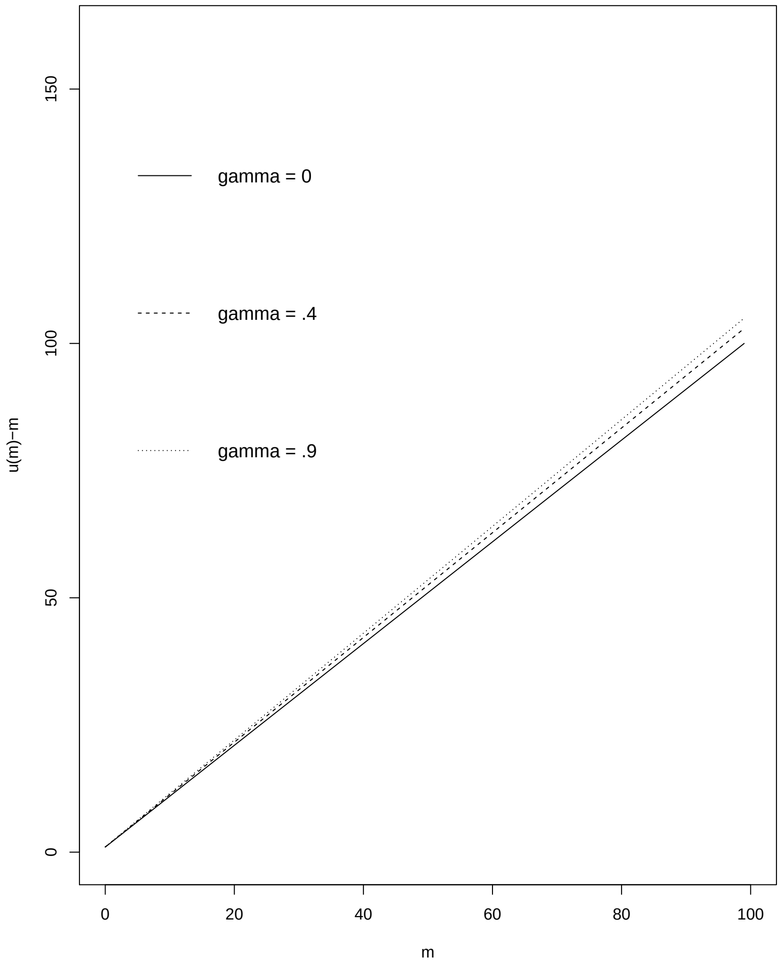

Figure 2: Faking level $u(m) - m$ versus m for different levels of γ. Here $\theta = .1$ and $\delta = 2$

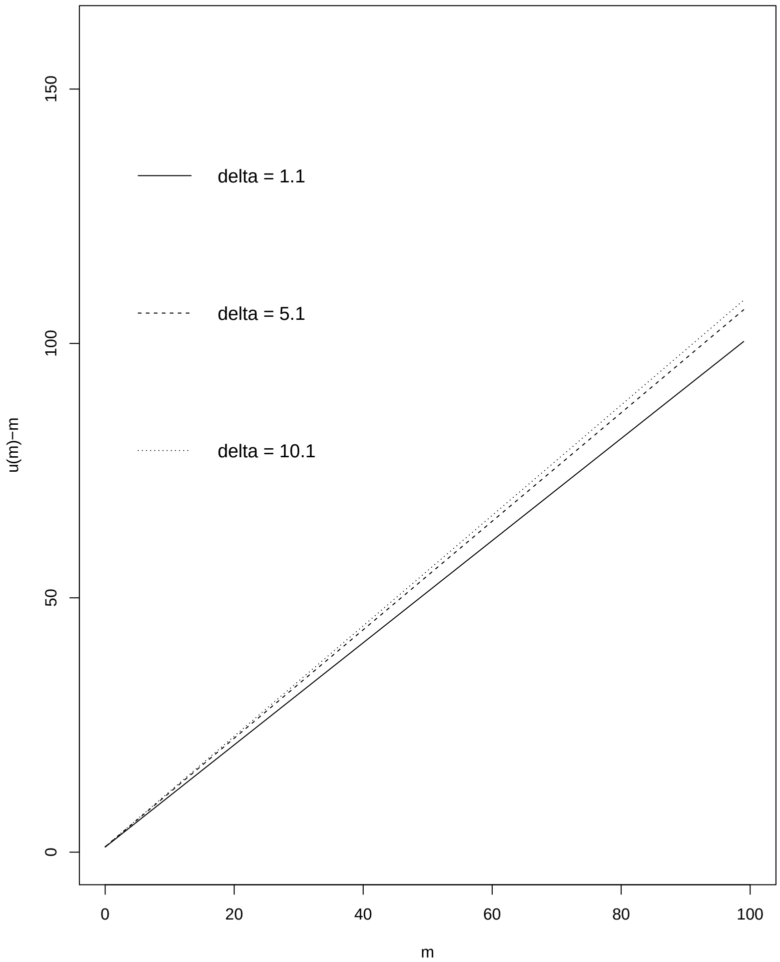

Figure 3: Faking level $u(m) - m$ versus m for different levels of δ. Here $\theta = .1$ and $\gamma = .4$

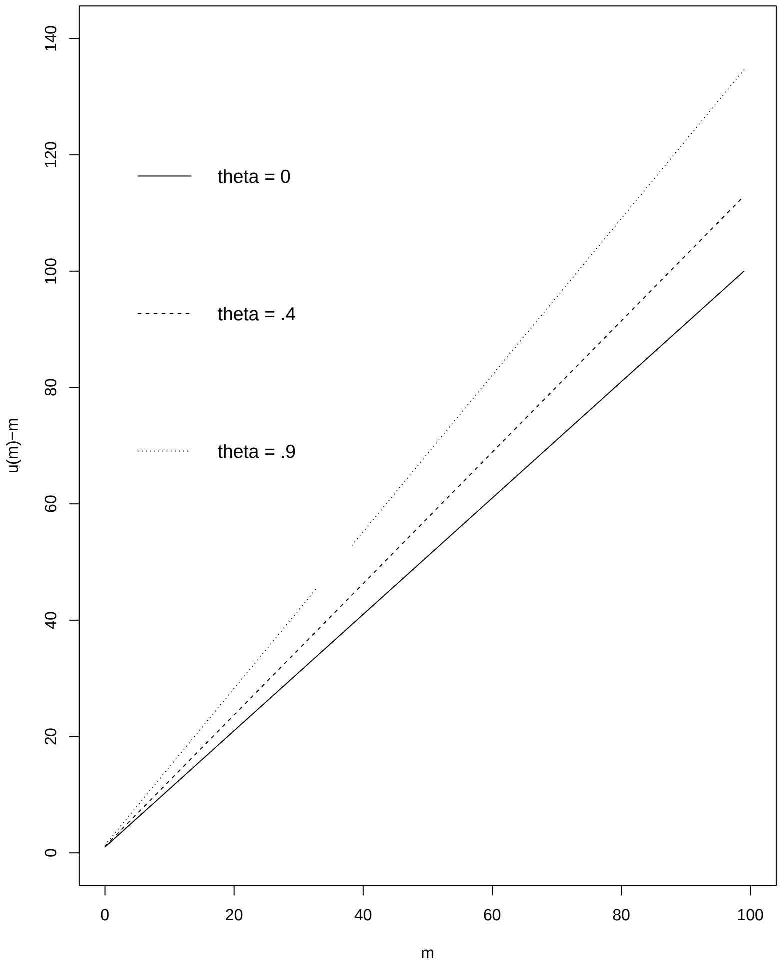

Figure 4: Faking level $u(m) - m$ versus m for different levels of θ. Here $\gamma = .4$ and $\delta = 2$