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Conditional VaR using EVT – Towards a planned margin scheme

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Abstract

This paper constructs a robust Value-at-Risk (VaR) measure for the Indian stock markets by combining two well-known facts about equity return time series — dynamic volatility resulting in the well-recognized phenomenon of volatility clustering, and non-normality giving rise to fat tails of the return distribution. While the phenomenon of volatility dynamics has been extensively studied using GARCH model and its many relatives, the application of Extreme Value Theory (EVT) is relatively recent in tracking extreme losses in the study of risk measurement. There are recent applications of Extreme Value Theory to estimate the unexpected losses due to extreme events and hence modify the current methodology of VaR. Extreme value theory (EVT) has been used to analyze financial data showing clear non-normal behavior. We combine the two methodologies to come up with a robust model with much enhanced predictive abilities. A robust model would obviate the need for imposing special *ad hoc* margins by the regulator in times of extreme volatility. A rule based margin system would increase efficiency of the price discovery process and also the market integrity with the regulator no longer seen as managing volatility.

Keywords: Dynamic VaR; GARCH; EVT

1. Introduction

Value at Risk (VaR) is a high quantile of the distribution of negative returns, typically the 95th or 99th percentile. It provides an upper bound for a loss that is exceeded only on a small

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proportion of occasions over a given time horizon. The VaR technique has undergone significant refinement since it originally appeared about a decade ago.

The first methodology used was the Analytic Variance–Covariance approach. It gained popularity as it was computationally efficient and the VaR could be calculated for a large portfolio using covariances among different securities in the portfolio. However, this methodology underestimated VaR most of the time as the actual return distributions exhibited heavier tails than that of the normal distributions the stock returns were assumed to follow.

To get around the difficulty of distributional assumption, historical simulation methodology was then employed. This method does not make any distributional assumptions, and the quantiles of the stock returns are calculated directly from the returns observed in the past. This ameliorates the problem of non-normality. However, this model assumes that the volatility of stock prices remains constant over long periods of time, and past estimates of volatility are good indicators of future volatilities.

The assumption of constant volatility is untenable as the phenomenon of volatility clustering is well documented in the literature. It has been observed across financial markets that large increases or decreases in prices are followed by large changes in either direction. Hence, stock price movements are characterized by periods of extreme volatility followed by periods of relative calm. In periods of extreme volatility, a static VaR would underestimate risk whereas it would be needlessly conservative during calm periods. Various authors have acknowledged the need to scale VaR measures by current volatility in some way (see Hull & White, 1998). The simplest dynamic risk model is the Random Walk model. The volatility forecast is based on any period in the past although, in practice, the time $t-1$ value is used to predict time t volatility. Extending this idea, various moving average methods have been developed — such as Historical Average, simple Moving Average, Exponential Smoothing and Exponentially Weighted Moving Average (EWMA). The Riskmetrics™ model is a procedure that uses the EWMA.

However, the above models have failed to provide a robust VaR estimate. This can be witnessed in the frequent imposition of *ad hoc* margins by the stock exchanges in India. The National Stock Exchange (NSE) imposes initial margins based on VaR estimates but imposes additional margins during periods of high volatility. The rationale of imposing additional margins is to safeguard the stock exchange system breaking down in the event of large-scale broker default in times of heightened volatility. But many commentators believe that such exogenous margin shock interferes with the price discovery process in the market and increase volatility in share prices rather than containing them. For example, many market participants believe that imposition of additional margins aided the largest single-day fall in Sensex on May 17, 2004. Brokers were unprepared to furnish extra *ad hoc* margins imposed by the exchange on account of increased volatility and had to liquidate their positions to meet the margin requirements. This only added to the selling pressure and further exacerbated the situation. This example underlines the importance of a rule-based margin base. If the margin setting mechanism were transparent and known to market players well in advance, they would be better prepared and would reduce forced liquidation of positions. The idea of increasing margins in times of high volatility is not wrong — the markets need to be saved from systemic failures. What is erroneous is the arbitrary nature of determination of margins as the VaR measures, used to calculate the base margins, are ineffective during increased volatility.

Hence, we require a dynamic VaR model that is robust during increased volatility and is known to participants before hand. The certainty and transparency of a rule based dynamic margin system would not impinge upon market efficiency while protecting the stock exchange from a default crisis.

The rest of the paper is divided into seven sections. Section 2 presents an overview of the appropriate volatility model namely, GARCH. Section 3 presents a broad outline of the Extreme Value Theory (EVT) used to model points in the tail of a distribution. Section 4 combines the two models to present a robust VaR measure. Section 5 analyzes the daily Nifty index return data using multiple VaR methodologies and discusses the back testing results of the various models. The expected shortfall, a coherent measure of risk, is discussed in Section 6. The policy implications are outlined in Section 7. Section 8 concludes.

2. Dynamic volatility — GARCH modeling

Most stock return series show a great deal of common structure. While the correlation of market returns is low, the serial correlation of squared returns at different time lags is high. This suggests that models based on the assumption that returns are independently and identically distributed need to be replaced by more sophisticated models using concepts of time series analysis.

The most widely used models for explaining this phenomenon are the dynamic volatility models, which take the form

$$X_t = \mu_t + \sigma_t Z_t \tag{1}$$

where σ_t is the volatility of the return on day t, μ_t is the expected return and X_t is the actual return. The randomness in the model comes through the stochastic variables Z_t, which are the residuals or the innovations of the process. We assume that the residuals Z_t are independently and identically distributed. By convention, these residuals are standardized i.e., they have mean equal to zero and variance equal to one, so that σ_t^2 is directly interpretable as the volatility of X_t. Although the structure of the model causes the X_t to be dependent, we assume X_t are identically distributed with unknown distribution function $F_X(x)$. This assumption is tenable if X_t is a stationary process. Therefore, it is of utmost importance to check for the stationarity of the return series before applying any dynamic volatility model.

ARCH (q) models were the first in a family of models to fit into this framework (see Bollerslev et al., 1992). The ARCH (q) model relates time t volatility to past squared returns up to q lags, with no predetermined relationships between any of the q dependencies. ARCH effects are detected on the basis of autocorrelation between the squared and/or absolute residuals of the return series. The ARCH (q) model was expanded to permit dependencies up to p lags of past volatility. The expanded model, GARCH (p, q) has been shown to be more parsimonious in various studies (see Poon & Granger (2003)). Therefore, GARCH models have become the most popular methodologies to describe dynamic volatility in financial time series.

Further enhancements of the GARCH models like EGARCH, RS-GARCH etc. have added complexity without significantly improving the forecasting performance of the model. In fact, GARCH (1,1) model has been found to perform satisfactorily for most stock return time series. Hence, we employ GARCH (1,1) to estimate dynamic volatility in this paper. The specification we use in this paper is:

$$\mu_t = \mu(\text{constant}) \tag{2}$$

$$\sigma_t^2 = w + \alpha(X_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2 \tag{3}$$

with w, α, $\beta > 0$ and $\alpha + \beta < 1$.

This specification has been shown to mimic many features of financial time series (see Poon & Granger (2003)). Also, this model can be interpreted easily as the variance of X_t being a weighted average of three components:

- a constant or unconditional volatility (w)
- yesterday's forecast (σ^2_{t-1})
- yesterday's news ($X_{t-1} - \mu_{t-1}$)

Verifying that the error series has constant mean and variance, and that there is no auto-correlation among various lags can test the validity of the model. The parameters of the GARCH model are estimated by the pseudo Maximum Likelihood procedure.

3. Modeling tails — EVT

A major criticism of the various VaR models has been that the higher percentiles, which are the points in the tails, are estimated using distributions that are designed to model tendency of stochastic variables that scatter around a central value — the mean. Such models are bound to underestimate the extreme events.

We need a distribution that exclusively models the higher percentiles. Extreme Value Theory plays a fundamental role in modeling the maxima of a stochastic variable. There are two ways of modeling extremes of a stochastic variable observed over a certain time horizon. The first approach divides the time horizon into blocks or periods and considers the maximum the variable takes in successive periods, for example months or years. These selected observations constitute the extreme events, also called block (or per-period) maxima.

In Fig. 1 (a), the observations X_2, X_5, X_7, and X_{11} represent the block maxima for four periods with three observations each. This is the traditional way of modeling extreme events and is used extensively in hydrology and other engineering applications.

However, this method is not particularly suited for financial time series because of volatility clustering. Due to this phenomenon, extreme events tend to follow one another. As the block maxima method takes into account only the maximum return in each period, a large number of relevant data points are excluded from the analysis. The second approach that utilizes data more efficiently takes into account points above a given high threshold. Therefore, the peak over threshold (POT) method has become the method of choice in financial applications. In Fig. 1 (b), the observations X_1, X_2, X_7, X_8, X_9 and X_{11} exceed the threshold u, and (a) (b) constitute extreme events. The distribution of exceedances above a threshold is based on theory developed by Fisher

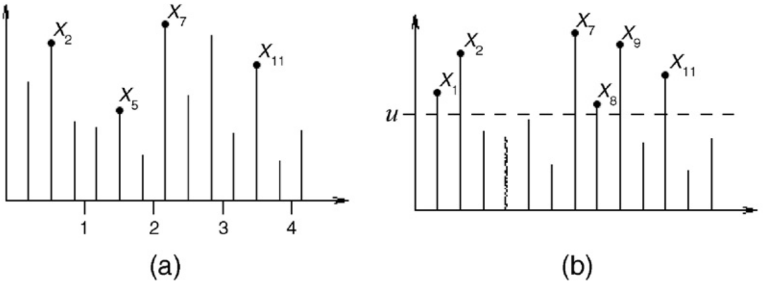

Fig. 1. (a) Block Maxima method, (b) Peak-Over-Threshold Method.

and Tippett (1928) that gives the limiting distribution of sample maxima provided the series has certain mathematical properties.

Building on this theory, Pickands (1975), Balkema and de Haan (1974) provided a result about conditional excess distribution function, which is stated in the following theorem.

Theorem: For a large class of underlying distribution functions the conditional excess distribution function $F_u\,(y)$, for a large value of u, is well approximated by

$$F_u(y) \cong G_{\xi,\beta}(y); u \to \infty, \tag{4}$$

where,

$$\begin{aligned} G_{\xi,\beta}(y) &= 1-(1+\xi y/\beta)^{-1/\xi}, \xi \neq 0 \\ &= 1-e^{-y/\beta}, \xi = 0 \end{aligned} \tag{5}$$

for $y \in [0, x_F - u]$ if $\xi \geq 0$ and $y \in [0, -\beta/\xi]$ if $\xi < 0$. $y = (x-u)$, u is the threshold; $x_F \leq \propto$ is the right endpoint of F. $G_{\xi\ ,\ \beta}$ is the so-called generalized Pareto distribution (GPD).

Fig. 2 depicts the shape of the GPD when ξ, called the shape parameter or tail index, takes a negative, a positive and a zero value. The scaling parameter β is kept equal to one. In general, one cannot fix an upper bound on financial losses. Therefore, the only relevant value of ξ for financial data is greater than zero. The method of estimation of u, ξ and β would be discussed in Section 5 when we present our data analysis.

3.1. Estimating VaR using EVT

$F_u(y)$ can also be expressed as

$$F_u(y) = (F(u+y) - F(u))/(1-F(u)) \tag{6}$$

for some underlying distribution F describing the entire time series X_t. Combining the above expression with the functional form of $F_u(y)$ written as described in the earlier paragraph, $F(x)$ can be written as

$$F(x) = (1-F(u))G_{\xi,\beta}(x-u) + F(u) \tag{7}$$

for $x > u$.

The aim of the above formulation is to construct a tail estimator for the underlying distribution $F(x)$. For this, we require an estimate of $F(u)$. An obvious candidate is empirical

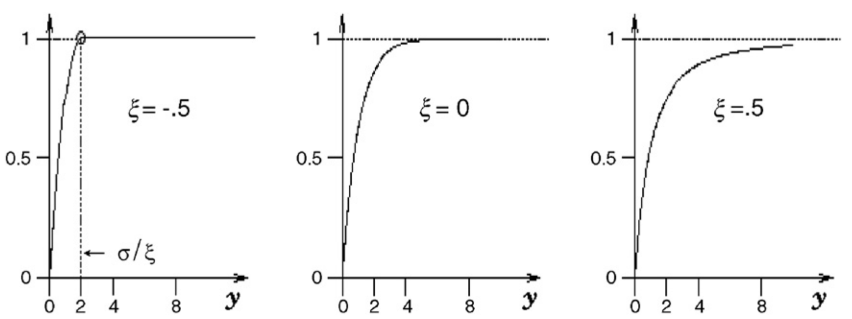

Fig. 2. Shape of a GPD for $\beta = 1$.

estimate $(n-N_u)/n$, where n is the total number of observations and N_u is the number of observations above the threshold, using the method of historical simulation (HS). A pertinent thought at this juncture would be why not using historical simulation to estimate the whole tail of $F(x)$ (i.e., for $x>u$). The reason is that the data are sparse above the threshold u and HS method does a poor job of simulation. We assume that there are sufficient points exceeding u to have a reasonable estimate of $F(u)$ but for higher confidence levels the estimates would be unreliable.

Putting together HS estimate of $F(u)$ and GPD, we arrive at the tail estimator

$$F(x) = 1-\frac{N_u}{n}\left(1+\xi\frac{x-u}{\beta}\right)^{-1/\xi} \tag{8}$$

for $x>u$. For a given probability, $q>F(u)$ the VaR estimate is calculated by inverting the tail estimation formula above to get (see Embrechts et al., 1997)

$$\mathrm{VaR}_q = u+\frac{\beta}{\xi}\left(\left(\frac{n}{N_u}(1-q)\right)^{-\xi}-1\right) \tag{9}$$

4. Combining the two models — dynamic risk management

In dynamic risk management, we are concerned with the conditional return distribution, $F_{t+1+\ldots\ldots+t+k|F_t(x)}(x)$, where $F_t(x)$ represents the history of the price process of X_t up to day t. In essence, we are trying to estimate the distribution of returns over the next $k\geq 1$ day given the present conditions. This is at variance with the unconditional or stationary distribution $F_{t+1+\ldots\ldots+t+k}(x)$. Here we are trying to estimate losses over a certain period in general, and not the risk we are exposed to, given the market conditions.

To reflect the dynamic nature of our VaR model, we introduce the notation, $\mathrm{VaR}^t{}_q(k)$. The subscript t signifies that it is a dynamic measure to be calculated at the close of day t; k denotes the time horizon. In this paper, we study one-day horizons. Therefore, we drop k. q denotes the quantile at which VaR is being calculated.

In calculating daily VaR estimates, it is now considered imperative to take into account the current volatility of the security. A high VaR value in periods of increased volatility appears less extreme when compared to the same value during calm periods. Many researchers have emphasized the need to scale the VaR estimates by some measure of *current* volatility and not an unconditional volatility for the entire period (see, for example, Hull & White, 1998). The GARCH family of models seems to be ideally suited for such modeling as described in an earlier section. The one- day VaR measure for the dynamic volatility model (GARCH) described earlier can be formulated as:

$$\mathrm{VaR}_q^t = \mu_{t+1}+\sigma_{t+1}\mathrm{VaR}(Z)_q \tag{10}$$

where $\mathrm{VaR}(Z)_q$ denotes the qth quantile of the noise variable Z_t.

This VaR measures accounts for volatility clustering in an elegant way and provides a measure of risk depending on the current market situation. A correct specification of the model makes the error terms *iid*, guaranteeing the theoretical soundness of $\mathrm{VaR}(Z)_q$ calculation. However, the deficiency of this model arises from the assumption that Z_t follows a known standard distribution. This assumption makes way for an easy calculation of $\mathrm{VaR}(Z)_q$. The problem with the assumption of conditional normality is that it tends to consistently underestimate the dynamic measure. The conditional distribution of GARCH models has been shown to have a heavier tail than that of a normal distribution.

Therefore, there is a need to augment the dynamic model with a correct formulation of tails. EVT appears to be an appropriate approach for modeling the tail behavior. But applying EVT to the random variable X_t is inappropriate as X_t is not independently and identically distributed. Thus, following the approach of McNeil and Frey (2000), we apply EVT to the noise variable Z_t rather than to X_t. We do not assume any functional form for $F(z)$. Instead, we apply the GPD tail estimation procedure described in the earlier section. The parameter and threshold estimation procedures of the GPD are described in the data analysis section. Hence, our estimation procedure for calculating a dynamic VaR at the end of day t using the return data of the last n days can be summarized as:

- An AR model with GARCH errors is fitted to the historical return data by pseudo maximum likelihood method. The errors (standardized residuals) of this model are extracted. If the model were correct, the error series Z_t would be realizations of the unobserved *iid* noise variables. The model is then used to calculate 1-day predictions of μ_{t+1} and σ_{t+1}.
- Apply EVT to standardized residuals. GPD tail estimation procedure is used to calculate VaR $(Z)_q$.
- VaR_q^t is calculated using the formula described in Eq. (10).

5. Parameter estimation and data analysis

We test the utility and validity of the improved dynamic VaR model using conditional EVT in the context of Indian markets by applying the model on daily returns of NSE Nifty index. The data consist of 2001 daily returns covering a period of 8 years from July 1, 1996 to June 30, 2004. Volatility and expected return forecasts are based on an AR(1) model with GARCH(1,1) errors. Specifically, the model is:

$$X_t = \mu_t + \sigma_t Z_t \tag{11}$$

$$\sigma_t^2 = w + \alpha(X_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2 \tag{12}$$

The model parameters were estimated using the Eviews© software package. The graph of the daily returns clearly depicted the phenomenon of volatility clustering, periods of extreme volatility followed by periods of relative calm.

The result of GARCH estimation procedure is given in Table 1.

All the coefficients of the volatility equation are significant. The Durbin Watson Statistic is 1.880697 implying that there is no autocorrelation among the residuals. Thus, the specification is tenable. The validity of the AR equation is verified from the correlogram. Correlations at all lags have been found to be insignificant implying that the return series is stationary. This satisfies another important criteria for the suitability of the GARCH model.

The GARCH specification has been shown to be appropriate. This takes care of volatility clustering. However, as argued in the previous sections, this is not enough as the descriptive statistics of the standard residuals clearly show that the conditional distribution has a heavier tail than that of a normal distribution.

The Jarque-Bera statistic is significant even at very low levels. Hence, we reject the null hypothesis that the standardized residuals follow a normal distribution. Kurtosis of 6.11 clearly indicates that the distribution has fat tails and the negative value of skewness indicates that the left tail (the tail of interest for VaR calculation) is particularly extreme. The results confirm our arguments in support of modeling the tail of the distribution separately (Table 2).

Table 1
GARCH estimation results

	Coefficient	Std. error	z-Statistics	Prob.
Return equation				
μ	0.00083	0.000341	2.424	0.015
Variance equation				
w	1.41E−05	1.94E−06	7.286	0.000
α	0.122142	0.009	12.850	0.000
β	0.833999	0.009	93.691	0.000

As described in earlier sections, we would employ the POT method using GPD for tail estimation. The first step in this modeling is to estimate the threshold for identifying the relevant tail region. A trade-off is involved in the choice of an appropriate threshold. A very high threshold leaves us with too few points for estimation. A low threshold level is a poor approximation as GPD is a limiting distribution for the case $u \rightarrow \propto$. Many researchers, McNeil (1997, 1999), employ a "high" enough percentile as the threshold. We employ a more systematic approach as described by Kluppelberg (2001). We define $e(u)=E[X-u|X>u]$ as mean excess function of X over the threshold u. For heavy tailed distributions, the mean excess function tends to infinity. Furthermore, for GPD, the mean excess function is a linear function given by:

$$e(u) = \frac{\sigma + \xi u}{1-\xi} \tag{13}$$

This is increasing if ξ is positive. As discussed earlier, ξ is always positive for financial time series. Thus, a possible choice of u is given by the value above which the observed mean excess function is approximately linear.

From Fig. 3, we get a value of threshold as 1.9. We get N_u (the number of points above the threshold) as 62, which is large enough to facilitate a good estimation.

The next step is the estimation of parameters, ξ and β of the GPD. The estimates can be obtained using the method of maximum likelihood. A more rigorous technique of Hill tail index estimator can also be used. It has been reported in the literature that this estimator

Table 2
Residual statistics

Series: RESID_NSE	
Sample 7/10/1996 to 6/30/2004	
Observations 2001	
Mean	−0.036495
Median	−0.017801
Maximum	6.740269
Minimum	−5.820477
Std. dev.	0.999513
Skewness	− 0.089278
Kurtosis	6.114087
Jarque-Bera	811.1905
Probability	0.000000

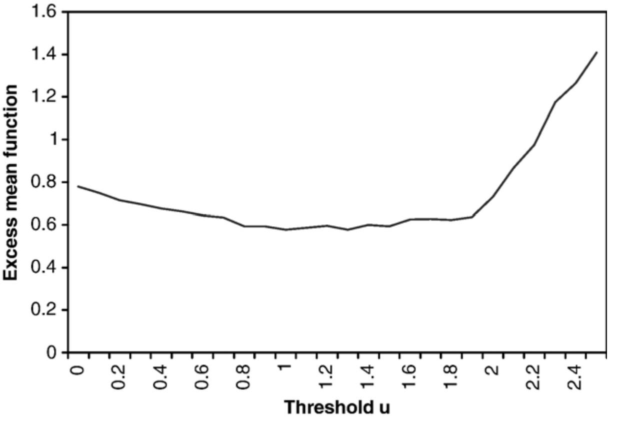

Fig. 3. Excess mean function.

performs better when compared to the maximum likelihood estimators. The Hill estimator for ξ is given by:

$$\xi = \left(\frac{1}{N_u}\right)\sum_{i=0}^{N_u-1}[\log X_{n-i+1} - \log X_{n-N_u}] \tag{14}$$

The National Institute of Standards and Technology, U.S. applies a correction to the Hill estimator for GPD. The steps for estimating the parameters are as follows:

- compute the quantities
- $M^{(r)} = \frac{1}{N_u}\sum_{i=0}^{N_u-1}[\log X_{n-i+1} - \log X_{n-N_u}]^r$ for $r = 1, 2$
- $\xi = M^{(1)} + 1 - \frac{1}{2[1-(M^{(1)})^2/M^{(2)}]}$
- $\beta = uM^{(1)}$

Following the above methodology the estimates of ξ and β are 0.4705 and 0.678 respectively. The estimates of the parameters from the two methods give similar results. In our calculations, we have used the modified Hill Parameter estimates because of their wide acceptability and they are shown to have better performance.

After specifying our model completely by estimating the parameters, we can now calculate the robust dynamic VaR estimates by using Eq. (10). We report the 97.5 percentile VaR. The value of $\mathrm{VaR}(Z)_{0.975}$ is found out be 2.42984. Using Eq. (10), our dynamic VaR specification for nifty returns is:

$$\mathrm{VaR}^t_{0.975} = \mu_{t+1} + 2.42984\sigma_{t+1} \tag{15}$$

where μ_{t+1} and σ_{t+1} are conditional GARCH estimates of mean and volatility.

The Fig. 4 shows the efficacy of our procedure. The VaR value changes dynamically to reflect market conditions. In periods of extreme volatility, the VaR value also increases and market safety is taken care of. Therefore, our model can form the basis of dynamic margin system that NSE employs without the need for *ad hoc* decisions. We formally test the superiority of our model versus the other static and dynamic formulations of VaR through a back testing procedure.

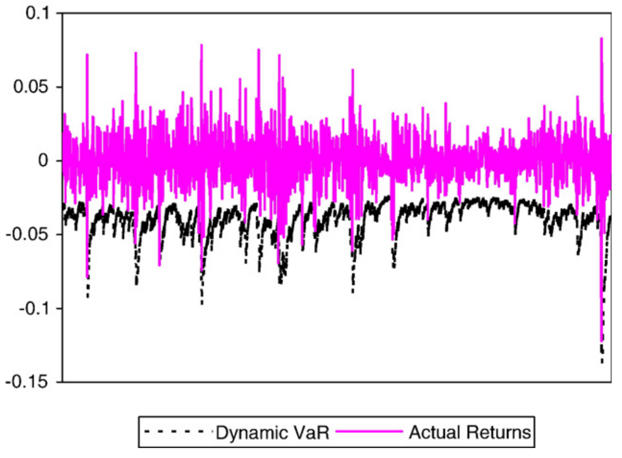

Fig. 4. Robust dynamic VaR.

5.1. Back testing the model

For back testing, we choose the first half of the calendar year 2004. This period witnessed extreme volatility in the stock markets due to politically uncertain environment and fears about the policy of the new Government. This period includes May 17, 2004 — the day when both Bombay Stock Exchange and the National Stock Exchange indices witnessed their largest one-day fall.

Comparing the estimates with the actual losses observed on the next day back tests a VaR estimation method. A violation occurs when the realized loss exceeds the estimated loss. Various dynamic and static methods of VaR estimation are compared by counting violations; tests of the violation counts based on the binomial distribution can show when a systematic underestimation or overestimation of VaR is taking place.

Table 3 reports the number of VaR violations that occurred during the testing period when estimating VaR with different methods. The *p*-values indicate the success of the estimation method based on hypothesis tests for the number of violations observed as compared to the expected number of violations. The expected number of violations is assumed to follow binomial distribution. A *p*-value less than 0.05 implies failure of the method at 5% significance level.

Table 3
Number of violations by different techniques

Length of series	Expected no. of violations	
125	3	
Method	No. of violations	*p*-value
Static normal	8	0.0033
Static HS	7	0.0109
Static EVT	5	0.0815
Dynamic normal	6	0.0317
Dynamic EVT	3	0.3528

The results clearly indicate that all estimates based on normality assumptions fare badly. This evidently underlines the importance of accounting for fat tails while modeling financial time series. The static measures also fail, reiterating the need to account for dynamic volatility in financial time series.

6. Expected shortfall estimation

APART from providing robust VaR measures, EVT based methods provide information that is more complete by enabling the estimation of severity of loss. The expected shortfall once the VaR limit is breached is given by

$$ES_q = \mathrm{VaR}_q + E[X - \mathrm{Var}_q | X > \mathrm{Var}_q] \tag{16}$$

The second term can be interpreted as the mean of the excess distribution $F_{\mathrm{VaR}_q}(y)$ over the threshold VaR_q. The EVT model for excess distribution above a given threshold is stable. If a higher threshold is taken, the excess distribution above the higher threshold is also a GPD with the same shape parameter but a different scaling parameter. An important corollary of this class of models is

$$F_{\mathrm{VaR}_q}(y) = G_{\xi,\beta+\xi(\mathrm{VaR}_{q-u})}(y) \tag{17}$$

This result allows us to estimate characteristics of the losses beyond VaR. The mean of the above distribution is given by $(\beta+\xi\,(\mathrm{VaR}_{q-u}))/(1-\xi)$. The expected shortfall is estimated as

$$ES_q = \mathrm{VaR}_q/(1-\xi) + (\beta-\xi u)/(1-\xi) \tag{18}$$

Based on calculations from our model, $ES(Z)_{0.975}$ is found to be 5.00665 using the above equation. This value is clearly large but this is to be expected in such heavy tailed data. Our dynamic ES specification for nifty returns is

$$ES^t_{0.975} = \mu_{t+1} + 5.00665\sigma_{t+1} \tag{19}$$

where μ_{t+1} and σ_{t+1} are conditional GARCH estimates of mean and volatility.

We report the actual losses and the shortfalls estimated by the model on the days the VaR limit was violated.

The Table 4 shows that the model gives conservative estimates of the losses. Risk management becomes more efficient now, as the decision-makers are aware of the magnitude of uncertainty about extreme events.

7. Policy implications and recommendations

The primary function of a market regulator is to institute policy frameworks that help in efficient price discovery and act as a watchdog against systemic events that distort market efficiency. To achieve this aim, the regulator has to come up with institutional mechanisms that are robust, transparent and accessible to market participants in advance. This helps in guiding and shaping participants′ behavior.

Table 4
Actual vs. estimated losses

Actual losses	Expected losses
−0.039736849	− 0.062628819
−0.078660844	− 0.091110778
−0.122377401	− 0.163266985

One of the major concerns for regulators across the globe is to avoid periods of extreme volatility in the markets as they impinge upon efficient price discovery and possible breakdown of the market mechanism itself due to heavy defaults.

In the absence of a robust, autonomous margin system that has an automatic feedback loop that factors in recent history to adjust margins, the regulators would be burdened with the unsavory task of 'managing' volatility. However, the *ad hoc* nature of this intervention often leads to more disruptions than solutions. Moreover, it disrupts the price discovery process.

In the Indian markets, we have a static margin system where the margins are fixed *a priori* and have no relation to the underlying volatility of the security. When volatility increases above a level where the regulators start feeling 'uncomfortable', margins are increased on an *ad hoc* basis with participants presented with fresh margin demands. Many a time positions have to be unwinded as the participants fail to meet the enhanced margin demands exacerbating the volatility situation rather than having a stabilizing impact.

Our research shows that a dynamic model that explicitly models extreme events and adjusts itself according to the price process is much more robust than any static method of arriving at a VaR estimate. A margin system based on this dynamic VaR would obviate the need for *ad hoc* interventions. Also, additional margin demands would no longer be a surprise but built into the systems of the participants.

Even after a year since May 17, 2004 when the Indian stock markets fell by more than 800 points, the Indian regulator is still investigating the events leading to the fault and its aftermath. Such investigations consume precious bandwidth without a systemic solution in sight. A dynamic VaR based system that autonomously increased margins with increasing volatility would have helped in stabilizing the markets more quickly. Possibly, the market fall would not have been as severe with the magnitude of forced unwinding much lower in a more certain rule based system.

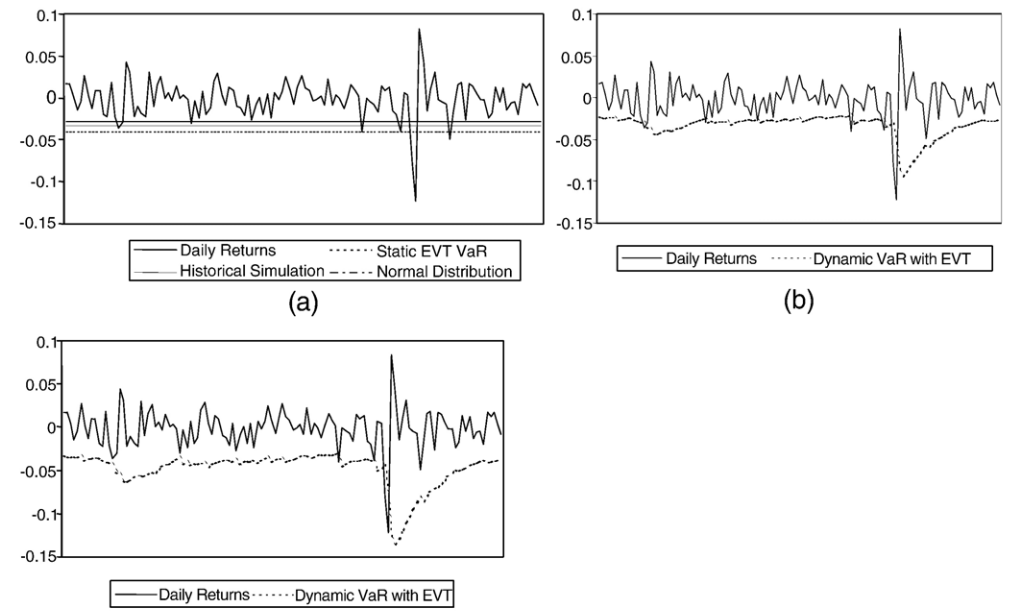

Fig. 5. VaR violations in (a) static models (b) dynamic model assuming normality and (c) dynamic model with tail estimation by GPD.

Our paper highlights how a robust, dynamic margin system can help increase market efficiency. Risk premiums tend to go down as the markets become more efficient. Lowering of discount rates help all market players in two ways — reducing the cost of capital and also lowering the hurdle rate for making fresh investments.

8. Conclusion

The main result of this paper is that dynamic VaR with tail estimation by Extreme Value Theory is the best method of estimating $\mathrm{VaR}^t{}_q$, at least in the Indian context. Static models are woefully inadequate in times of extreme volatility. As seen in Fig. 5, VaR violations are particularly extreme for static methods. Thus, not only are the violations more frequent, the severity is also more leading to a totally inadequate risk measure. Dynamic measures with normality assumptions are also not good enough as they underestimate VaR.

Markets in India stand to benefit significantly by adoption of the enhanced VaR model for calculation of daily margins. This VaR models addresses the twin concerns of safety and efficiency. During periods of large volatility, the dynamic nature of the model would lead to appropriate increases in VaR measures to ensure safety. As the margins would be rule based now, there would be no surprises in terms of extra margin requirements on an *ad hoc* basis. This would lead to an increase in market efficiency and a better price discovery process. Moreover, regulators would no longer be seen as managing volatility — a function neither required nor desirable of a regulator.

This paper is concerned with a one-day horizon for VaR calculation. A simple rule analogous to the square root of time rule for scaling VaR to multiple periods under the assumption of normal distribution of returns is not available in this method. A Monte Carlo simulation for estimating the possible future path of dynamic volatility is required. The estimation of multiple periods VaR will be taken up separately.

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Further reading

Bhattacharyya, M., & Ritolia, G. (2005, May). *EVT enhanced dynamic VaR — A rule based margin system, presented at the 3rd financial markets Asia–Pacific conference, Sydney, Australia*.