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Labour Search with Endogenous Outside Option

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Abstract

We incorporate the extended version of Nash bargaining proposed by Vartiainen (2007) in a standard labour search and matching model to endogenously determine the outside option of workers along with their wages. We find that the optimal outside option of a worker under this framework is zero and this equilibrium maximizes social welfare when the economy is constrained efficient.

1 Introduction

The search and matching model (Pissarides (2000)) is a widely accepted paradigm for understanding equilibrium unemployment. This model has two main components. First, unemployed workers and vacant firms meet each other and the flow of new matches is determined by a matching function. Second, upon getting matched, workers and firms bargain over wages. This wage setting process is typically modeled as a cooperative bargaining game with Nash bargaining as the solution concept. A key determinant of the bargaining outcome is the outside option of the workers and is usually assumed to be exogenous.

In this paper, we seek to endogenize the choice of outside option of workers, which we interpret as unemployment insurance. We do this by adopting the extended version of the Nash bargaining model, introduced in Vartiainen (2007), in the benchmark labour search model. Vartiainen (2007) extends the Nash bargaining framework to determine both the solution and the outside option simultaneously. He shows that, this can be achieved by maximizing the Nash product with respect to both the surplus division and the outside

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options. Our main finding is that the optimal outside option of a worker in this framework is zero.

We also study the efficiency properties of our search equilibrium. When the outside option is interpreted as home production, it is well-known from Hosios (1990) that the search equilibrium is efficient if and only if the elasticity of matching function is equal to the bargaining power of workers. But, when the outside option is interpreted as unemployment insurance, l'Haridon et al. (2013) show that in addition to the Hosios' condition, unemployment benefits need to be zero for search equilibrium to be efficient. Since the unemployment insurance is endogenously determined to be zero in our model, the Hosios condition completely characterizes the efficiency of our equilibrium. In other words, incorporating the generalized Nash bargaining framework does not add any new inefficiency beyond what is due to the mismatch between the bargaining power and the elasticity of the matching function.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the extended Nash bargaining framework. In section 4, we study the equilibrium of the search model. Finally, in section 5 we discuss the efficiency properties of our search equilibrium and conclude in Section 6.

2 The Model

We consider a benchmark search and matching model discussed in Pissarides (2000) with no aggregate uncertainty. There is a unit measure of risk neutral workers and a continuum of firms. The total number of firms is determined in the equilibrium by a free entry condition. Both firms and workers maximize their lifetime income and they discount any future income at rate r.

Suppose there is a fraction u of unemployed workers and a fraction v of vacant firms in the economy. The total number of new matches formed (m) is governed by a constant returns to scale matching function $m(u, v)$. The rate at which unemployed workers get employed is given by

$$\frac{m(u, v)}{u}$$

Since $m(u, v)$ is constant return to scale, we have

$$\frac{m(u,v)}{u} = m\Big(1, \frac{v}{u}\Big) = q(\theta).$$

where $\theta = v/u$ is called the labour market tightness. Similarly the rate at which vacancies get filled is

$$\frac{m(u,v)}{v} = \frac{v}{u} m\Big(1, \frac{v}{u}\Big) = \theta q(\theta).$$

Once a match is formed, the match persists until the firm and the worker separate exogenously with a probability s. Existing match has a productivity of y, out of which the worker receives a wage w while the firm earns a profit of $y - w$. Equating the transitions into $(s(1-u))$ and out of $(\theta q(\theta) u)$ unemployment, we derive the steady state unemployment as

$$u = \frac{s}{s + \theta q(\theta)} \tag{1}$$

Continuation Values

Let J be the value enjoyed by a matched firm while the value of an unfilled vacancy is V. Similarly the value of an employed worker is given by E and that of an unemployed worker is U. The value functions are defined as follows.

$$rU = z + \theta q(\theta)(E - U) \tag{2}$$

$$rE = w + s(U - E) \tag{3}$$

$$rV = -\gamma + q(\theta)(J - V) \tag{4}$$

$$rJ = y - w + s(V - J) \tag{5}$$

where z is the outside option of the worker and γ is the cost of posting vacancy for an unmatched firm. Usually z is assumed to be exogenous and w is obtained as a solution to Nash bargaining for a given z. In this paper, we determine both z and w endogenously by employing extended Nash bargaining solution proposed by Vartiainen (2007).

3 Bargaining

Usually wages are determined by Nash bargaining proposed by Nash Jr (1950), where wage is chosen to maximize the Nash product for a given outside option. Vartiainen (2007) shows that the disagreement outcome can be obtained endogenously along with the wage by maximizing the Nash product with respect to both wage and the outside option. He further shows that, this extended framework is the only solution that satisfies the axioms of Pareto-optimality, symmetry, independence of irrelevant alternatives and scale invariance, analogous to Nash Jr (1950).

Under this new framework, the bargaining problem gets modified to

$$(w^*, z^*) = \arg\max_{w,z} (E - U)^{\phi} (J - V)^{1-\phi} \tag{6}$$

$E - U$ is the surplus enjoyed by an employed worker and $J - V$ is the surplus of a matched firm, while $\phi \in [0, 1]$ denotes the bargaining power of the worker. w^* and z^* are the resulting wage and outside option of this bargaining problem.

4 Search Equilibrium

Usually, the search equilibrium consists of market tightness θ, wage w, and unemployment u. Our equilibrium, in addition to these objects also includes worker's outside option z obtained during bargaining. In equilibrium, the firms enter until the value of a new vacancy (V) is driven down to zero. Substituting the free entry condition ($V = 0$) into the value functions (4) and (5), we obtain the job creation condition:

$$\frac{\gamma}{q(\theta)} = \frac{y - w}{r + s} \tag{7}$$

The job creation condition (7) along with the first-order conditions of the bargaining problem (6) can be solved together to obtain search equilibrium.

Proposition 1. *The equilibrium wage and outside option are*

$$\begin{aligned} w^* &= \phi(y + \gamma\theta) \\ z^* &= 0 \end{aligned}$$

Proof. In Appendix. ■

We endogenously solve for worker's outside option z and find that it is zero in equilibrium. Next, we discuss the efficiency properties of this search equilibrium.

5 Efficiency

It is well known that, decentralized equilibrium in the presence of search frictions is not efficient. An additional vacant firm makes workers better off but makes other firms worse off. Similarly, an extra unemployed worker makes the firms better off but makes other workers worse off. Following l'Haridon et al. (2013), the benevolent social planner chooses θ and u to maximize the total production given by

$$\max_{\theta,u} \Gamma = (1-u)y - \gamma\theta u \tag{8}$$

subject to equation (1). The planner maximises total output net of search costs. Both wages (w) and unemployment benefits (z) do not feature in the planner's objective function as we assume that the planner cares only about the total output and not the distribution. Under this assumption, social welfare is captured by the total output Γ.

As shown in Hosios (1990), a necessary condition for efficiency of a search equilibrium with Nash bargaining is to have the elasticity of matching function with respect to unemployment equal the bargaining power of the worker. This is popularly called as the Hosios' condition, which translates to $\eta = \phi$ in our setup. An economy satisfying the Hosios' condition is said to be constrained efficient and we obtain the following result.

Proposition 2. *Nash bargaining with endogenous outside option maximizes social welfare when the economy is constrained efficient.*

Proof. In Appendix. ∎

As l'Haridon et al. (2013) shows, an additional requirement for maximizing social welfare is to set $z = 0$ (Proposition 2, pp.46).[1] A typical search equilibrium with Nash bargaining need not maximize social welfare as there is no a priori reason for unemployment benefits, z, to be zero. By adopting the endogenous Nash bargaining solution, we

[1]Under a different setup, Acemoglu and Shimer (1999) also show that an economy with risk-neutral workers achieves maximal output when unemployment insurance is zero.

show that the optimal choice of z is zero and thus the decentralized equilibrium is welfare maximizing if the economy is constrained efficient.

6 Conclusion

By adopting the extended Nash bargaining solution of Vartiainen (2007), we endogenize the choice of outside option in the classical labour search model. Our main finding is that (*i*) the optimal outside option of a worker, i.e. unemployment benefits is zero and (*ii*) the inefficiency in the decentralized equilibrium is solely driven by the mismatch between the elasticity of the matching function and the bargaining power of the workers. Although we study the baseline model of labour search, we believe that the insights from our model will be useful to study richer frameworks such as those incorporating aggregate uncertainty, endogenous job destruction and on-the-job search. We leave these interesting questions for future research.

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Appendix

A Proof of Proposition 1

A.1 Choice of Wage w

Following Vartiainen (2007), we choose the outside option z by maximizing the Nash product:

$$(w^*, z^*) = \arg\max_{w,z} (E-U)^{\phi}(J-V)^{1-\phi}$$

A.1.1 Wage Bargaining

We first solve for w, given z. Later, we choose z that maximizes the Nash product.

$[w]$:

$$(1-\phi)(E-U)\frac{\partial(J-V)}{\partial w} + \phi(J-V)\frac{\partial(E-U)}{\partial w} = 0 \tag{9}$$

We next solve for the partial derivatives in the above equation. Using value function (3):

$$\begin{aligned} r(E-U) &= w + s(U-E) - rU \\ (r+s)(E-U) &= w - rU \end{aligned}$$

Simplifying

$$(E-U) = \frac{w - rU}{r+s} \tag{10}$$

Differentiating w.r.t w

$$\frac{\partial(E-U)}{\partial w} = \frac{1}{r+s} \tag{11}$$

Similarly, from equation (5):

$$\begin{aligned} r(J-V) &= y - w + s(V-J) - rV \\ (r+s)(J-V) &= y - w - rV \end{aligned}$$

Simplifying

$$J - V = \frac{y - w - rV}{r + s} \tag{12}$$

Differentiating w.r.t. w

$$\frac{\partial(J - V)}{\partial w} = \frac{-1}{r + s} \tag{13}$$

Plugging equations (11) and (13) back into equation (9):

$$(1 - \phi)(E - U)\Big(\frac{-1}{r + s}\Big) + \phi(J - V)\Big(\frac{1}{r + s}\Big) = 0$$

This implies

$$\phi(J - V) = (1 - \phi)(E - U) \tag{14}$$

Plugging in equations (10) and (12),

$$\phi(y - w - rV) = (1 - \phi)(w - rU) \tag{15}$$

A.1.2 Free Entry ($V = 0$)

Plugging free entry into value function (4)

$$J = \frac{\gamma}{q(\theta)} \tag{16}$$

Substituting free entry into value function (5)

$$J = \frac{y - w}{r + s} \tag{17}$$

Combining the above 2 equations

$$J = \frac{\gamma}{q(\theta)} = \frac{y - w}{r + s} \tag{18}$$

Plugging free entry into equation (15):

$$w = \phi y + (1 - \phi)rU \tag{19}$$

A.1.3 Solving for rU

From equation (2)

$$rU = z + \theta q(\theta)(E - U)$$

Substituting eqn (14)

$$rU = z + \theta q(\theta)\frac{\phi}{1-\phi}J$$

Substituting eqn (18)

$$rU = z + \frac{\phi}{1-\phi}\gamma\theta \tag{20}$$

A.1.4 Wage Equation

Plugging this back into equation (19), we obtain the bargained wages

$$w = \phi y + (1-\phi)z + \phi\gamma\theta \tag{21}$$

A.2 Choice of Outside Option z

We substitute the wage equation back into the Nash product to solve for bargained z.

A.2.1 Nash Product

The Nash product is

$$N = (E-U)^{\phi}(J-V)^{1-\phi}$$

Substituting equations (10) and (12)

$$N = \Big(\frac{w - rU}{r+s}\Big)^{\phi}\Big(\frac{y-w}{r+s}\Big)^{1-\phi}$$

Substituting expressions for w and rU into Nash product:

$$N = \phi^{\phi}(1-\phi)^{1-\phi}\left[\frac{y - (z + \frac{\phi}{1-\phi}\gamma\theta)}{r+s}\right] \tag{22}$$

This is a function in z and θ. Differentiating w.r.t. z:

$$\frac{dN}{dz} = \frac{-\phi^{\phi}(1-\phi)^{1-\phi}}{r+s}\left[1 + \frac{\phi}{1-\phi}\gamma\frac{d\theta}{dz}\right] \tag{23}$$

A.2.2 Calculating $\frac{d\theta}{dz}$

Solving for θ from job creation condition (18)

$$\frac{\gamma}{q(\theta)} = \frac{y-w}{r+s} \tag{24}$$

Using wage equation (21)

$$y - w = (1-\phi)\Big[y - \Big(z + \frac{\phi}{1-\phi}\gamma\theta\Big)\Big]$$

Assuming Cobb-Douglas matching function,

$$q(\theta) = \theta^{-\alpha}$$

Plugging these into equation (24)

$$\frac{\gamma}{\theta^{-\alpha}} = \frac{(1-\phi)\Big[y - \Big(z + \frac{\phi}{1-\phi}\gamma\theta\Big)\Big]}{r+s}$$

Rewriting

$$\theta^{\alpha} = \frac{(1-\phi)\Big[y - \Big(z + \frac{\phi}{1-\phi}\gamma\theta\Big)\Big]}{\gamma(r+s)}$$

Total differentiating the above expression with respect to both θ and z

$$\alpha\theta^{\alpha-1}d\theta = \frac{-(1-\phi)}{\gamma(r+s)}\Big[dz + \frac{\phi}{1-\phi}\gamma d\theta\Big]$$

Rearranging

$$\frac{d\theta}{dz} = \frac{\frac{-(1-\phi)}{\gamma}}{[\alpha\theta^{\alpha-1}(r+s)+\phi]} \tag{25}$$

Two observations:

1. $\frac{d\theta}{dz} < 0$
2. $\left|\frac{\phi}{1-\phi}\gamma\frac{d\theta}{dz}\right| < 1$

A.2.3 Bargained z

Substituting (25) back into equation (23)

$$\frac{dN}{dz} = \frac{-\phi^{\phi}(1-\phi)^{1-\phi}}{r+s}\left[1 - \frac{\phi}{\alpha\theta^{\alpha-1}(r+s)+\phi}\right] \tag{26}$$

We get,

$$\frac{dN}{dz} < 0 \tag{27}$$

Thus,

$$z^* = 0 \tag{28}$$

Nash product is a decreasing function of z and hence $z = 0$ maximizes the Nash product.

B Proof of Proposition 2

We first derive the planner's solution and then show that our decentralized equilibrium matches the planner's solution, thus proving the efficiency result.

B.1 Planner's Problem

Following l'Haridon et al. (2013), the planner chooses θ and u to maximize social welfare given by

$$\max_{\theta,u} \Gamma = (1-u)y - \gamma\theta u \tag{29}$$

subject to equation (1). Differentiating (29) w.r.t. θ, we get

$$(1-\eta)y - \left[\frac{s+\eta\theta q(\theta)}{q(\theta)}\right]\gamma = 0 \tag{30}$$

where η is the elasticity of matching function w.r.t. unemployment. Equations (1) and (30) solve for θ and u that maximises social welfare.

B.2 Decentralized Equilibrium

We derive the decentralized equilibrium condition by combining job creation condition (18) with wage equation (21) and outside option (28)

$$(1-\phi)y - \left[\frac{s + \phi\theta q(\theta)}{q(\theta)}\right]\gamma = 0 \tag{31}$$

Comparing (30) and (31), decentralized equilibrium matches planner's solution when $\eta = \phi$. Thus, search equilibrium with endogenous Nash bargaining maximises social welfare as long as the economy is constrained efficient.